MAC 2311 CALCULUS WITH ANALYTIC GEOMETRY ONE
TEST 4

Name KEY Score $\overline{53}$

1. The velocity function (in meters per second) for a particle moving along a straight line is given by $v(t) = -(t-4)^2 + 4$ for $1 \le t \le 6$.

   a) Set-up and solve an integral to find the displacement of the particle during the indicated time period. (3 points)

$$\int_1^6 v(t)\,dt = \int_1^6 -t^2+8t-12\,dt = \left[-\frac{1}{3}t^3+4t^2-12t\right]_1^6 = 0 - -\frac{25}{3} = \frac{25}{3}$$

$\approx$ 8.33 meters

   b) Set-up, but DO NOT solve, two integrals whose sum is the total distance traveled by the particle during the indicated time period. (4 points)

$$\left|\int_1^2 v(t)\,dt\right| + \left|\int_2^6 v(t)\,dt\right| = \left|-\frac{7}{3}\right| + \left|\frac{32}{3}\right| = 13 \text{ meters}$$

2. The velocity of a vehicle is measured every five seconds and recorded in the table below. Estimate the distance traveled by the vehicle during the first thirty seconds in three ways as outlined below.

time (sec)	0	5	10	15	20	25	30
velocity (ft./sec)	25	31	35	43	47	46	41

a) Calculate R_6, i.e. use right-hand endpoints with 6 rectangles. (2 points)

$$\Delta x = \frac{30-0}{6} = 5$$

$$5(31+35+43+47+46+41) = 1215 \text{ ft.}$$

b) Calculate L_2, i.e. use left-hand endpoints with 2 rectangles. (2 points)

$$\Delta x = \frac{30-0}{2} = 15$$

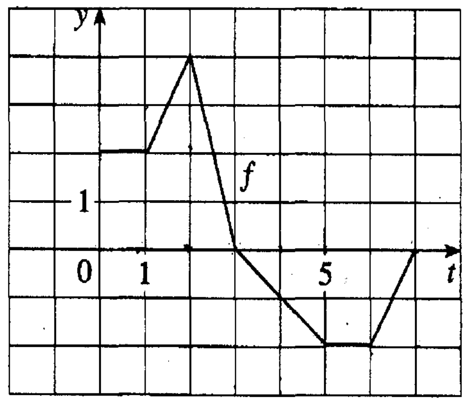
$$15(25+43) = 1020 \text{ ft.}$$

c) Calculate M_3, i.e. use midpoints with 3 rectangles. (2 points)

$$\Delta x = \frac{30-0}{3} = 10$$

$$10(31+43+46) = 1200 \text{ ft.}$$

3. Let $g(x) = \int_0^x f(t)dt$ where $0 \le t \le 7$ and f is the function whose graph is below.



- Calculate $g(2)$ and $g(6)$. (2 points)

$g(2)=5$

$g(6)=3$

- Find the interval where g is increasing. (1 points)

Inc on $(0,3)$

- Find the intervals where g is concave up and concave down. (2 points)

concave-up on $(1,2) \cup (6,7)$

concave-down on $(2,3) \cup (3,5)$

- The function g has only ONE local extreme value. Find the x AND y coordinates of this point. (2 points)

Local max. at $(3,7)$

4. Use the Fundamental Theorem of Calculus to compute $\int_0^{\pi}(4\sin(2x) - 3\cos(2x))\,dx$.

(3 points)

$u = 2x$

$\frac{du}{2} = dx$

$$\frac{1}{2}\int_0^{2\pi}(4\sin u - 3\cos u)\,du$$

$$= \frac{1}{2}\Big[-4\cos u - 3\sin u\Big]_0^{2\pi}$$

$$= \frac{1}{2}\left[-7 - {}^{-}7\right]$$

$$= \boxed{0}$$

5. Use a substitution to compute the indefinite integral $\int x^2 \sec^2(1+2x^3)\,dx$. (4 points)

$u = 1 + 2x^3$

$\frac{du}{6} = x^2\,dx$

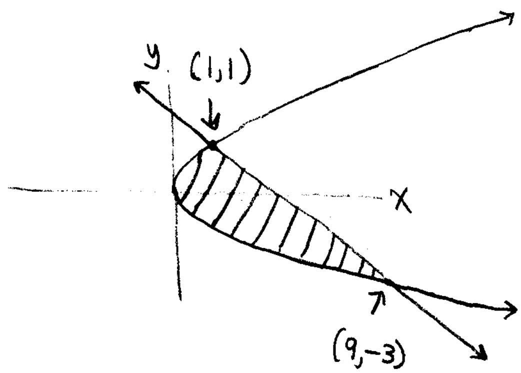
$$\frac{1}{6}\int \sec^2 u\,du = \frac{1}{6}\tan u + c$$

$$= \frac{1}{6}\tan(1+2x^3) + c$$

6. Set-up and <u>solve</u> ONE integral that gives the area bounded between the curves whose equations are given below. (6 points)

$$2y + x = 3$$

$$x = y^2$$



$$A = \int x_R - x_L \, dy$$

$$= \int_{-3}^{1} (3-2y) - (y^2) \, dy$$

$$= \left[3y - y^2 - \frac{1}{3}y^3\right]_{-3}^{1} = \frac{5}{3} - -9 = \boxed{\frac{32}{3}}$$

7. Fill in the blanks with the upper and lower limits as well as the integrand for the integral appearing in the equation below. (3 points)

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^3}{n^3}\right)\frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$a =$ 0

$$x_i = a + i\Delta x = a + \frac{i}{n} = \frac{i}{n} \Rightarrow a = 0$$

$b =$ 1

$$\Delta x = \frac{b-a}{n} = \frac{1}{n} \Rightarrow b-a = 1 \Rightarrow b = 1$$

$f(x) =$ x^3

8. Approximate the integral $\int_{-2}^{3} \frac{1}{x^2+1}\,dx$ using TWO rectangles and left-hand endpoints.

(3 points)

$$\Delta x = \frac{3 - -2}{2} = 2.5$$

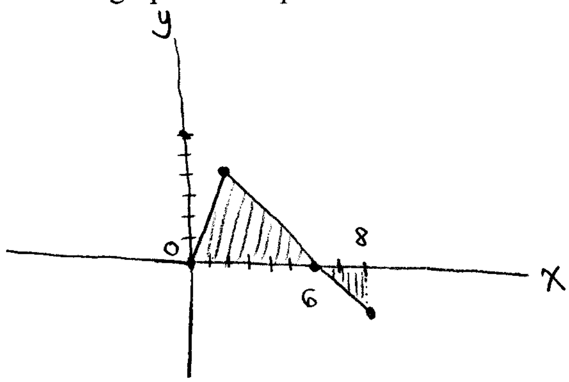
$$A = 2.5\left[f(-2) + f(0.5)\right] \quad \text{where } f(x) = \frac{1}{x^2+1}$$

$$A = 2.5(0.2 + 0.8)$$

$$= \boxed{2.5}$$

9. Calculate $\int_0^8 g(x)\,dx$ where $g(x) = \begin{cases} 2x & \text{for } 0 \le x < 2 \\ -x+6 & \text{for } x \ge 2 \end{cases}$

Include a sketch of the graph of this piecewise defined function in your response. (6 points)



$$\int_0^8 g(x)dx = \int_0^6 g(x)dx + \int_6^8 g(x)dx$$

$$= \frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2$$

$$= \frac{1}{2}(6)(4) + \frac{1}{2}(2)(-2)$$

$$= 12 + -2$$

$$= \boxed{10}$$

10. Calculate each of the following using the Fundamental Theorem of Calculus and/or the rules and properties of integrals. (8 points)

a) $\frac{d}{dx}\int_{-5}^{x} \sin(t^2)\,dt = \sin(x^2)$

b) $\int\left(y^{1/3} - \frac{8}{y^2} + 3^{-1}\right)dy = \int\left(y^{1/3} - 8y^{-2} + \frac{1}{3}\right)dy$

$= \frac{3}{4}y^{4/3} + 8y^{-1} + \frac{1}{3}y = \frac{3}{4}y^{4/3} + \frac{8}{y} + \frac{y}{3} + c$

c) $\int(5p^3 - 2)(p+4)\,dp = \int(5p^4 + 20p^3 - 2p - 8)\,dp$

$= p^5 + 5p^4 - p^2 - 8p + c$

d) $\int_0^2 \sqrt{4-x}\,dx = \frac{a-\sqrt{b}}{3}$ (Find the numbers a and b that fit the equation)

$u = 4 - x$

$-du = dx$

$-\int_4^2 \sqrt{u}\,du = \int_2^4 \sqrt{u}\,du = \left[\frac{2}{3}u^{3/2}\right]_2^4 = \frac{16}{3} - \frac{2}{3}\sqrt{8}$

$= \frac{16 - 2\sqrt{8}}{3}$

$= \frac{16 - \sqrt{32}}{3}$ so $a = 16$, $b = 32$