MAC 2311 CALCULUS WITH ANALYTIC GEOMETRY ONE
TEST 3

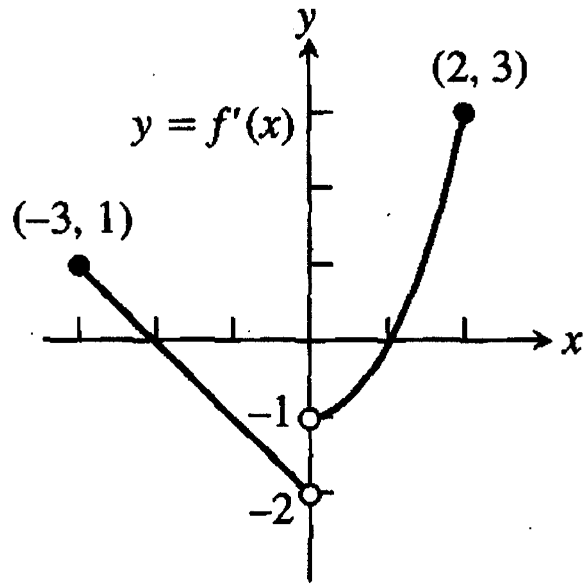
Name KEY   Score 55

Directions: Answer each question showing ALL work for full credit.

1. Use calculus and algebra to find ALL critical numbers of the function $f(x) = x^{1/3}(5x-8)$ (6 points)

2. The function $p(x) = x^3 - 3x^2 + 3x + 1$ is a polynomial. We know that all polynomial functions are continuous and differentiable everywhere. In particular, $p(x)$ is continuous on the closed interval $[-1, 5]$ and differentiable on the open interval $(-1, 5)$. This satisfies the hypotheses of the Mean Value Theorem. Use calculus and algebra to find a number c that satisfies the conclusion of the theorem. (6 points)

3. The function f is continuous on [-3, 2] and the graph of $y = f'(x)$ is shown below. Answer the following questions.



a) On what intervals is f increasing and decreasing? (2 points)

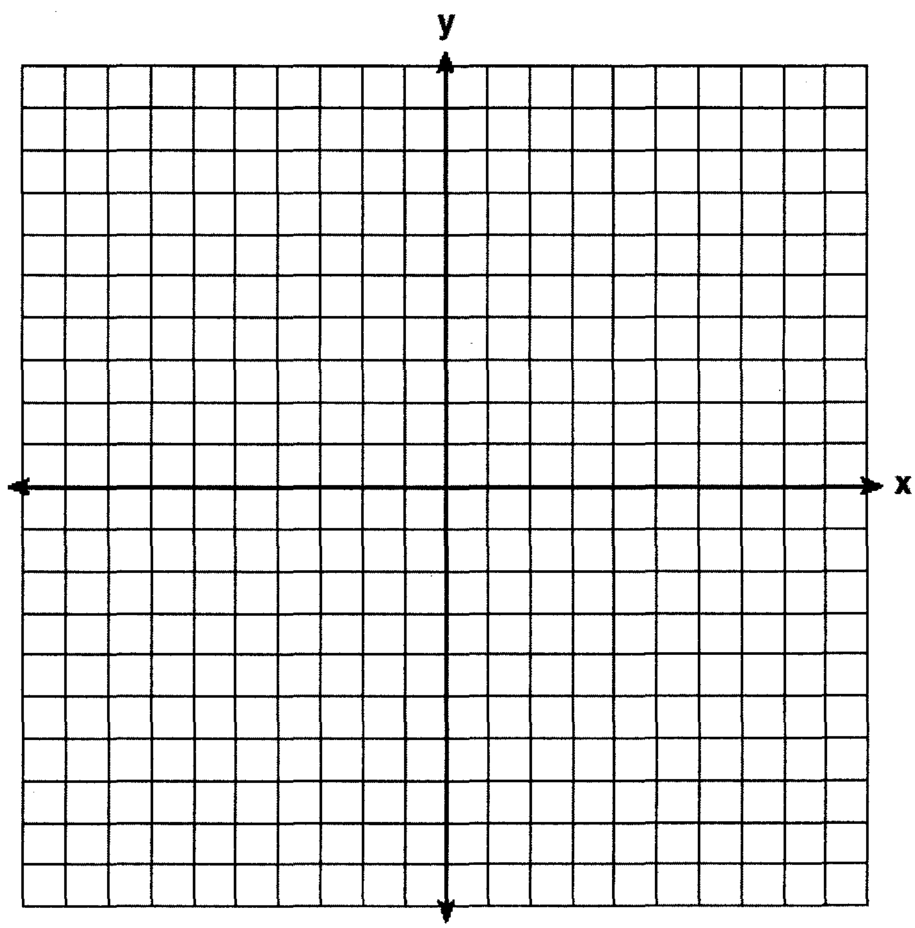
b) At what values does f have a local maximum and minimum? (2 points)

c) On what intervals is f concave upward and downward? (2 points)

d) What are the x-values of the inflection point(s)? (1 points)

4. Sketch the graph of a function that satisfies the given conditions. (8 points)

- $f(0) = 0$ and $f(2) = 3$ and $f(4) = 1$
- $f'(2) = 0$ and $f'(0)$ is undefined
- $f'(x) > 0$ if $0 < x < 2$
- $f'(x) < 0$ if $x > 2$
- $f''(x) < 0$ if $0 < x < 4$
- $f''(x) > 0$ if $x > 4$
- $\lim_{x \to \infty} f(x) = 0$
- $f(-x) = -f(x)$ for all values of x. (This means f is an odd function).



5. Evaluate the following limit using algebraic techniques. (6 points)

$$\lim_{x \to \infty} \left(\sqrt{9x^2 + x} - 3x \right)$$

6. Use the closed interval method to find **both** the absolute minimum and absolute maximum values of the function $f(x) = \dfrac{\cos(x)}{2+\sin(x)}$ on the interval $[0, 2\pi]$. Express your final answers in exact form which means **do not** use decimals. (Hint: use a famous trig identity) (8 points)

7. American Airlines requires that the total outside dimensions (length + width + height) of a carry-on bag not exceed 45 inches. Suppose you want to carry on a bag whose length is twice its height. What is the largest volume bag of this shape that you can carry on an American flight? Use calculus and the first derivative test to answer this question. (8 points)

8. A particle is moving with an acceleration that is given below as a function of time (in seconds). Find the position (in meters) of the particle after 3 seconds given the initial conditions. Be sure to show the anti-derivatives in your work. (6 points)

$$a(t) = t - 2$$

$$s(0) = 1 \text{ and } v(1) = 3$$

-------------------- END OF TEST --------------------

TEST 3

1. $y = x^{1/3} \cdot (5x-8)$

$y' = \frac{1}{3}x^{-2/3}(5x-8) + 5x^{1/3}$

$y' = \dfrac{5x-8}{3x^{2/3}} + \dfrac{5x^{1/3}}{1}$

$y' = \dfrac{5x-8+15x}{3x^{2/3}} = \dfrac{20x-8}{3x^{2/3}} \rightarrow$ set $20x-8=0$ and $3x^{2/3}=0$

obtain $x = \frac{2}{5}$ and $x=0$

2. $p(x) = x^3 - 3x^2 + 3x + 1$

$p'(x) = 3x^2 - 6x + 3$

There is a number c in $(-1,5)$ with $p'(c) = \dfrac{P(5)-P(-1)}{5-^-1} = \dfrac{66-^-6}{6} = 12$

we need to solve $3c^2 - 6c + 3 = 12$

$3c^2 - 6c - 9 = 0$

$3(c^2 - 2c - 3) = 0$

$3(c-3)(c+1) = 0$

$c-3=0$ or $c+1=0$

$c=3$ or $\cancel{c=-1}$ ← not in the open interval $(-1,5)$.

③ a) f is increasing on $(-3,-2) \cup (1,2)$

f is decreasing on $(-2,1)$

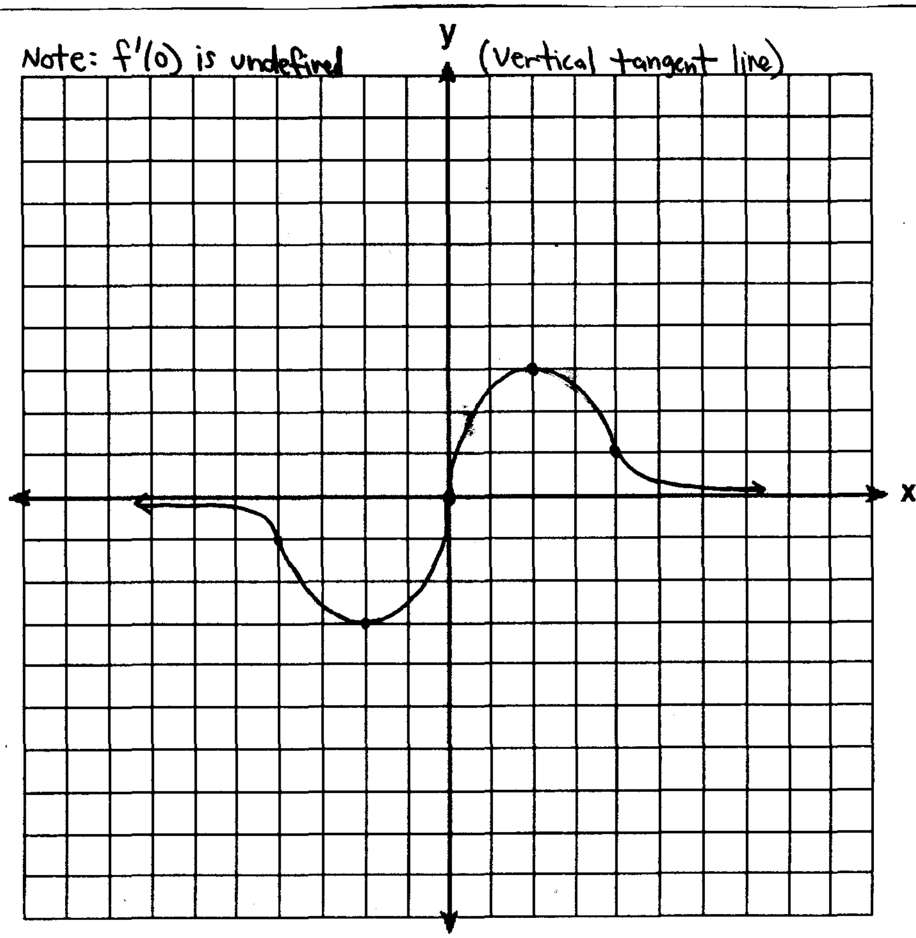
b) Local max at $x=-2$
Local min at $x=1$

c) f is concave up on $(0,2)$
f is concave down on $(-3,0)$

d) Inflection Point at $x=0$

④

Note: $f'(0)$ is undefined (Vertical tangent line)



$$\sqrt{9x^2+x} - 3x =$$

5 $$= \frac{\sqrt{9x^2+x} - 3x}{1} \cdot \frac{\sqrt{9x^2+x} + 3x}{\sqrt{9x^2+x} + 3x}$$

$$= \frac{9x^2+x-9x^2}{\sqrt{9x^2+x} + 3x}$$

$$= \frac{x}{\sqrt{9x^2+x} + 3x}$$ Since $x \to \infty$ we may assume $x = \sqrt{x^2}$

$$= \frac{\frac{x}{x}}{\frac{\sqrt{9x^2+x}}{\sqrt{x^2}} + \frac{3x}{x}}$$ divide top and bottom by x.

$$= \frac{1}{\sqrt{9+\frac{1}{x}} + 3}$$

so $$\lim_{x\to\infty}\left(\sqrt{9x^2+x} - 3x\right) = \lim_{x\to\infty}\left(\frac{1}{\sqrt{9+\frac{1}{x}} + 3}\right) = \frac{1}{\sqrt{9+0} + 3} = \frac{1}{6}$$

(6) $f(x) = \dfrac{\cos x}{2+\sin x}$ so $f'(x) = \dfrac{(2+\sin x)(-\sin x) - (\cos x)(\cos x)}{(2+\sin x)^2}$

$$f'(x) = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$f'(x) = \frac{-2\sin x - 1}{(2+\sin x)^2}$$

since $(2+\sin x)^2$ is never zero the critical values must occur when $-2\sin x - 1 = 0$ or when $\sin x = -\frac{1}{2}$

set $-2\sin x - 1 = 0$

$\sin x = \frac{-1}{2}$

$x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$ on $[0, 2\pi]$

$\frac{7\pi}{6}$: Quad III, $\frac{11\pi}{6}$: Quad IV

closed interval method:

$$f\left(\frac{7\pi}{6}\right) = \frac{\frac{-\sqrt{3}}{2}}{2 + \frac{-1}{2}} = \frac{\frac{-\sqrt{3}}{2}}{\frac{3}{2}} = \frac{-\sqrt{3}}{3}$$

$$f\left(\frac{11\pi}{6}\right) = \frac{\frac{\sqrt{3}}{2}}{2 + \frac{-1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3}$$

	x	$f(x)$	
end pts.	0	$\frac{1}{2}$	
	2π	$\frac{1}{2}$	
critical pts.	$\frac{7\pi}{6}$	$\frac{-\sqrt{3}}{3}$	← abs. min
	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{3}$	← abs. max

⑦ $l+w+h=45 \longrightarrow h = 45 - l - w$

Constraints $= 45-(2w)-w$ since $l=2w$

$l = 2w$ $\qquad h = 45-3w$

substitute

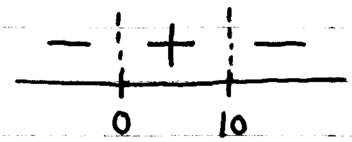
$V = l \cdot w \cdot h$

$= (2w)w \cdot h$

$= 2w^2(45-3w)$

$V = 90w^2 - 6w^3$

sign chart



solve $\quad V' = 180w - 18w^2 = 0$

$18w(10-w) = 0$

$w=0$ or $w=10$ (critical pts)

Since the 1st derivative changes sign from (+) to (−) at $w=10$, this critical number yields a local max.

Max. Volume is $V = 90(10)^2 - 6(10)^3$

$= 9000 - 6000$

$= 3000 \text{ in}^3$

8 $a(t) = t - 2$

$v(t) = \frac{1}{2}t^2 - 2t + c$ but $v(1) = \frac{1}{2} - 2 + c = 3$ so $c = \frac{9}{2}$

$s(t) = \frac{1}{6}t^3 - t^2 + \frac{9}{2}t + D$ but $s(0) = D = 1$

$s(t) = \frac{1}{6}t^3 - t^2 + \frac{9}{2}t + 1$

$s(3) = \frac{1}{6}(3)^3 - (3)^2 + \frac{9}{2}(3) + 1 = 10$ meters