MAC 2311 CALCULUS WITH ANALYTIC GEOMETRY ONE
TEST 2

Name KEY Score $\overline{58}$

Directions: Answer each question, showing ALL your work for full credit.

1. Calculate the derivative of each of the following functions. (8 points)

$$f(x) = \pi^2 x + \underbrace{\frac{3}{x^{-4}}}_{3x^4} + \frac{1}{2}x^6 - \underbrace{\pi^2}_{\text{constant}}$$ (Reduce any fractions in the final answer)

$f'(x) = \pi^2 + 12x^3 + 3x^5 - 0$

$g(x) = 10\sqrt[5]{x} = 10x^{\frac{1}{5}}$ (Express the final answer in radical notation)

$g'(x) = \frac{10}{5}x^{-4/5} = \frac{2}{\sqrt[5]{x^4}}$

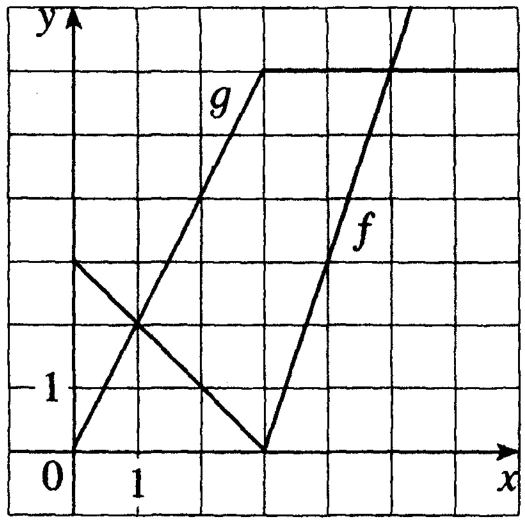
$h(x) = \tan x + \cos x$

$h'(x) = \sec^2 x - \sin x$

$k(x) = \sec(x^2)$

$k'(x) = 2x\sec(x^2)\cdot\tan(x^2)$

2. Let f and g be functions whose graphs are shown below. Suppose that $P(x) = f(x) \cdot g(x)$ and $C(x) = f(g(x))$. Calculate, step by step, the values of $P'(2)$ and $C'(2)$. (8 points)



$P'(2) = f'(2) \cdot g(2) + f(2) \cdot g'(2) = (-1)(4) + (1)(2) = -4 + 2 = \boxed{-2}$

$C'(2) = f'(g(2)) \cdot g'(2) = f'(4) \cdot g'(2) = (3) \cdot (2) = \boxed{6}$

3. a) Use the derivative to find the equation of the tangent line to the graph of $y = \frac{8x}{x+1}$ at $x = 1$.

(6 points)

$$y' = \frac{8(x+1) - 8x(1)}{(x+1)^2} = \frac{8}{(x+1)^2}$$ ← Plug in $x=1$ get $m=2$

when $x=1$, $y=4$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x-1)$$

$$\boxed{y = 2x + 2}$$

b) Calculate the 2[nd] derivative of $y = \frac{8x}{x+1}$. (2 points)

$$\boxed{y'' = \frac{-16}{(x+1)^3}}$$

$$y' = \frac{8}{(x+1)^2} = 8(x+1)^{-2}$$ from part a)

$$y'' = -16(x+1)^{-2-1} \cdot (1)$$

↑ chain rule

4. A particle moves along a straight line according to the equation $s = t^3 - 9t^2 + 15t + 10$ where $t \geq 0$, t is measured in seconds and s in feet.

a) What is the velocity of the particle after 3 seconds. (1 points)

★★ $v = 3t^2 - 18t + 15$

$v(3) = -12 \text{ ft/s}$

b) When is the particle at rest? (2 points)

Solve $3t^2 - 18t + 15 = 0$

$t^2 - 6t + 5 = 0$

$(t-5)(t-1) = 0$

So $t = 1, 5$

c) When is the particle moving in the positive direction? (2 points)

solve $3t^2 - 18t + 15 > 0$

$[0,1) \cup (5, \infty)$

d) When is the particle slowing down? (2 points)

★★ $a = 6t - 18 = 0$ implies $t = 3$

	[0,1)	(1,3)	(3,5)	(5,∞)
v	+	−	−	+
a	−	−	+	+

0 1 2 3 4 5

Sign chart

Slow down when signs are different for v and a

on $[0,1) \cup (3,5)$

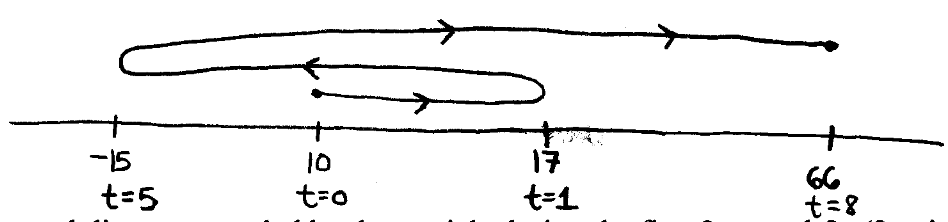
Switch answers.

e) What is the total displacement of the particle during the first 8 seconds of travel? (1 point)

$7 + 32 + 81 = 120 \text{ ft}$

(0,1) (1,5) (5,8) distance

time intervals



f) What is the total distance traveled by the particle during the first 8 seconds? (2 points)

$66 - 10 = 56 \text{ ft}$

S(8) S(0) displacement

5. Find $\frac{dy}{dx}$ by implicit differentiation given the equation $x^2 + xy - y^2 = 4$. Express your final answer as a SINGLE fraction. (6 points)

$$2x + \overbrace{(1)(y) + (x)(y')}^{\text{Product Rule for } (xy)'} - 2y \cdot y' = 0$$

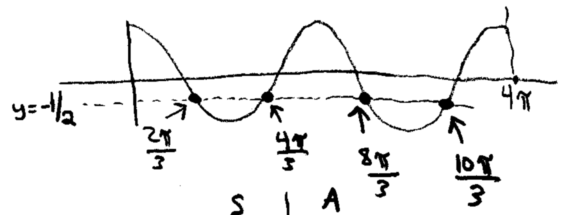
solve for y'

$$xy' - 2yy' = -2x - y$$

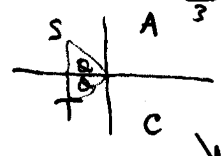
$$y' = \frac{dy}{dx} = \frac{-2x-y}{x-2y}$$

6. For what values of x on the interval $[0, 4\pi]$ does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent? Note: you are going around the unit circle twice. (6 points)

$$f'(x) = 1 + 2\cos x = 0$$



$$\cos x = \frac{-1}{2}$$



$$x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

reference ∡ is $\theta = \frac{\pi}{6}$

$$\frac{2\pi}{3} = \pi - \frac{\pi}{6}$$

$$\frac{8\pi}{3} = \frac{2\pi}{3} + 2\pi$$

Quadrant II

where $\cos x$ is negative.

$$\frac{4\pi}{3} = \pi + \frac{\pi}{6}$$

$$\frac{10\pi}{3} = \frac{4\pi}{3} + 2\pi$$

Quadrant III

7. The derivative of $f(x)=\dfrac{x}{\sqrt{x^2+4}}$ can be written in the form $f'(x)=\dfrac{k}{(x^2+4)^m}$ where k and m are real numbers. Find the exact values for k and m. (6 points)

$$f'(x)=\frac{\sqrt{x^2+4}-\dfrac{\cancel{2}x^2}{\cancel{2}\sqrt{x^2+4}}}{x^2+4}=\frac{x^2+4-x^2}{(x^2+4)^{3/2}}=\frac{4}{(x^2+4)^{3/2}}$$

so $k=4$, $m=3/2$

More detailed solution:

chain rule

$$f'(x)=\frac{\dfrac{\sqrt{x^2+4}}{1}(1)-x\dfrac{1}{2\sqrt{x^2+4}}\cdot 2x}{\left(\sqrt{x^2+4}\right)^2}$$ (Quotient Rule)

$$=\frac{\dfrac{\sqrt{x^2+4}}{1}-\dfrac{x^2}{\sqrt{x^2+4}}}{x^2+4}$$ add the top 2 fractions together. (Cross mult. method)

x^2's cancel

mult. by reciprocal

$$=\frac{\dfrac{(x^2+4)-x^2}{\sqrt{x^2+4}}}{\dfrac{x^2+4}{1}}=\frac{4}{\sqrt{x^2+4}}\cdot\frac{1}{x^2+4}=\frac{4}{(x^2+4)^1\sqrt{x^2+4}}$$

$$=\frac{4}{(x^2+4)^{1+\frac{1}{2}}}$$

$$=\frac{4}{(x^2+4)^{3/2}}$$

8. The area of a square is increasing at a rate of 10 cm^2 per second. At what rate is the perimeter of the square increasing when the area of the square is 225 cm^2? (6 points)

$A = x^2$ differentiate

$\frac{dA}{dt} = 2x \cdot \frac{dx}{dt}$

$P = 4x$ diff.

$\frac{dP}{dt} = 4 \cdot \frac{dx}{dt}$

Know

$\frac{dA}{dt} = 10 \frac{cm^2}{s}$

$A = 225\ cm^2$ so $x = 15$ cm

ie. $\sqrt{225} = 15$

$10 = 2(15) \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{3} \frac{cm}{s}$

plug in

$\frac{dP}{dt} = 4\left(\frac{1}{3}\right) = \frac{4}{3} \frac{cm}{s}$

$\frac{dP}{dt} = 1.3\overline{3} \frac{cm}{s}$

-----------------------------End of Test------------------------------