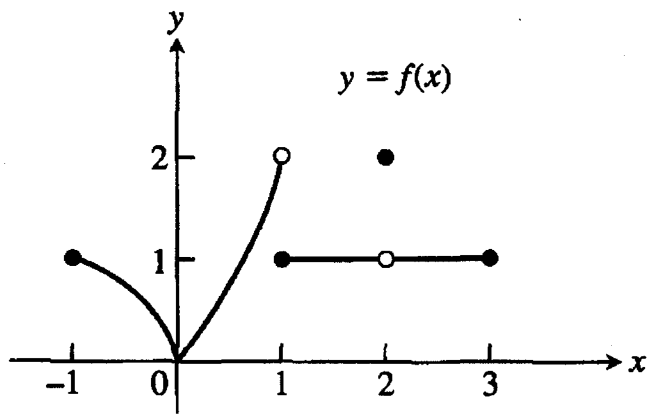
MAC 2311 CALCULUS WITH ANALYTIC GEOMETRY I
TEST 1

Name KEY　　　　　　Score $\overline{56}$

Directions: Answer each question showing ALL work for full credit.

1. Use the graph of the function below to determine each statement as TRUE or FALSE. No work is required for this particular problem. (8 points)



T $\lim_{x\to -1^+} f(x) = 1$

F $\lim_{x\to 2} f(x) = 2$ $\lim_{x\to 2} f(x) = 1$

F $\lim_{x\to a} f(x)$ exists for all numbers a in the interval $(-1,3)$ except for 1 and 2.
$\lim_{x\to 2} f(x)$ exists.

F $\lim_{x\to 1^-} f(x) = f(1)$
$\lim_{x\to 1^-} f(x) = 2 \neq f(1)$

T $\lim_{x\to 1} f(x)$ does not exist

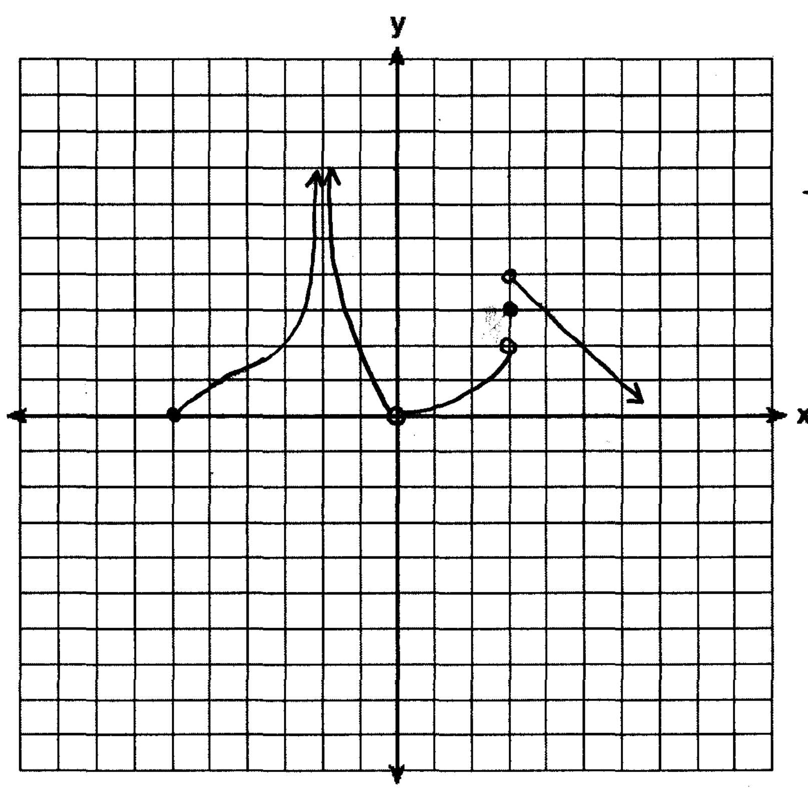
T $f(x)$ is continuous at $x = \frac{3}{2}$

T $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

F $f(x)$ is continuous from the right at $x = 2$
$\lim_{x\to 2^+} f(x) = 1 \neq f(2)$

2. Sketch the graph of a function that satisfies all of the conditions below. (8 points)

- $\lim_{x \to 3^+} f(x) = 4$
- $\lim_{x \to 3^-} f(x) = 2$
- $\lim_{x \to -2} f(x) = \infty$
- $\lim_{x \to 0} f(x) = 0$
- $f(3) = 3$
- $f(0)$ is undefined
- $f(x) = -x + 7$ for $x > 3$



This is ONE Possibility.

3. The definition for the derivative of a function f at $x = 0$ is given below

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}.$$

a) Use this definition to calculate the slope of the tangent line to the graph of the function $f(x) = x^2 + 2x$ at $x = 0$. (3 points)

$$f'(0) = \lim_{h \to 0} \frac{h^2 + 2h - 0}{h} = \lim_{h \to 0} \frac{h(h+2) - 0}{h} = \lim_{h \to 0} (h+2) \overset{\text{D.S.P}}{=} 0 + 2 = \boxed{2}$$

b) You are given that $g(x) = \dfrac{1}{x-4}$ and $g'(5) = -1$. Use this information to find the equation of the line tangent to the graph of $y = g(x)$ at $x = 5$. (3 points)

$$g(5) = \frac{1}{5-4} = 1$$

Use $y - y_1 = m(x - x_1)$

$y - 1 = -(x - 5)$

$\boxed{y = -x + 6}$

4. A cardiac monitor is used to compile the number of heartbeats for a patient. The values in the table below show the cumulative number of heartbeats H after t minutes. Here H is a function of time. For example $H(38) = 2661$.

P

t (min.)	36	38	40	42	44	46
H	2530	2661	2806	2948	3080	3202

a) If P is the point $(42, 2948)$ on the graph of H, find the slope of the secant line PQ where Q is the point on the graph when $t = 46$. (2 points)

$$m_{sec} = \frac{3202-2948}{46-42} = \boxed{63.5}$$

b) Give the <u>best</u> possible estimate for the slope of the tangent line at P by averaging the slopes of two secant lines. (2 points)

$$m_{right} = \frac{3080-2948}{44-42} = 66 \qquad \frac{66+71}{2} = \boxed{68.5}$$

$$m_{left} = \frac{2948-2806}{42-40} = 71$$

c) Write a sentence or two, using layman's terms, describing what the equation $H'(42) = 68$ means in context of this problem. Be specific and including units of measurement. (2 points)

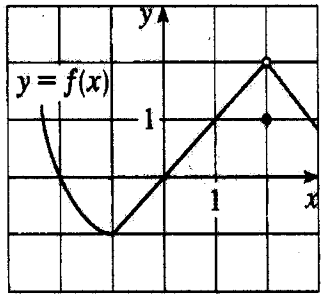
42 minutes after a patient is connected to the cardiac monitor his/her heartrate is 68 beats per minute.

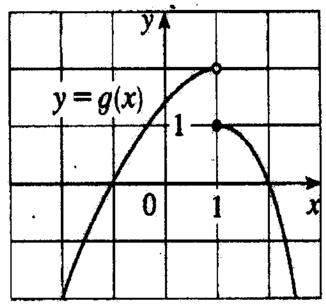
d) According to the table, during which two minute interval did the patient have the highest heart rate? (2 point)

$38 < t < 40$

The heartrate averages out to be 72.5 bpm

5. The graphs of f and g are given below. Use them to evaluate each limit, if it exists. If the limit does not exist write DNE in the space provided. (6 points)





$\lim_{x\to 2}[f(x)+g(x)] =$ 2 $\quad \lim_{x\to 1}[f(x)+g(x)] =$ DNE $\quad \lim_{x\to 0}[f(x)g(x)] =$ 0

$\lim_{x\to -1}\dfrac{f(x)}{g(x)} =$ DNE $\quad \lim_{x\to 2} x^3 f(x) =$ 16 $\quad \lim_{x\to 1}\sqrt{3+f(x)} =$ 2

6. Use algebra to compute both of the following limits, if they exist. (8 points)

$$\lim_{x\to -1}\left(\frac{x^2+3x+2}{x^2-x-2}\right)$$

$$= \lim_{x\to -1} \frac{(x+1)(x+2)}{(x+1)(x-2)}$$

$$= \lim_{x\to -1}\left(\frac{x+2}{x-2}\right)$$

$$\overset{\text{D.S.P}}{=} \frac{-1+2}{-1-2}$$

$$= \boxed{\frac{-1}{3}}$$

$$\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}\right)$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$ conjugate

$$= \lim_{x\to 0} \frac{1+x-1}{x(\sqrt{1+x}+1)}$$

$$= \lim_{x\to 0} \frac{x}{x(\sqrt{1+x}+1)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1}$$

$$\overset{\text{D.S.P}}{=} \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

7. Use limits to find the constant c that makes g continuous everywhere. (4 points)

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

$$\lim_{x \to 4^-} g(x) = \lim_{x \to 4^-} (x^2 - c^2) \overset{\text{D.S.P}}{=} 16 - c^2$$

$$\lim_{x \to 4^+} g(x) = \lim_{x \to 4^+} (cx + 20) \overset{\text{D.S.P}}{=} 4c + 20$$

$$g(4) = 4c + 20$$

we need to solve $16 - c^2 = 4c + 20$

$$c^2 + 4c + 4 = 0$$

$$(c+2)^2 = 0$$

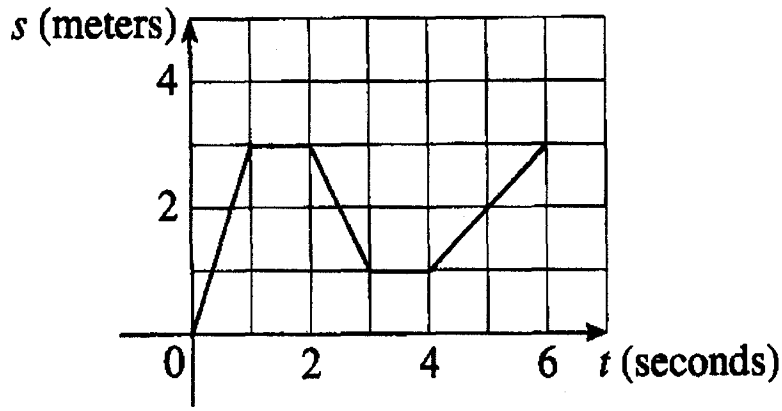
$$c + 2 = 0$$

$$\boxed{c = -2}$$

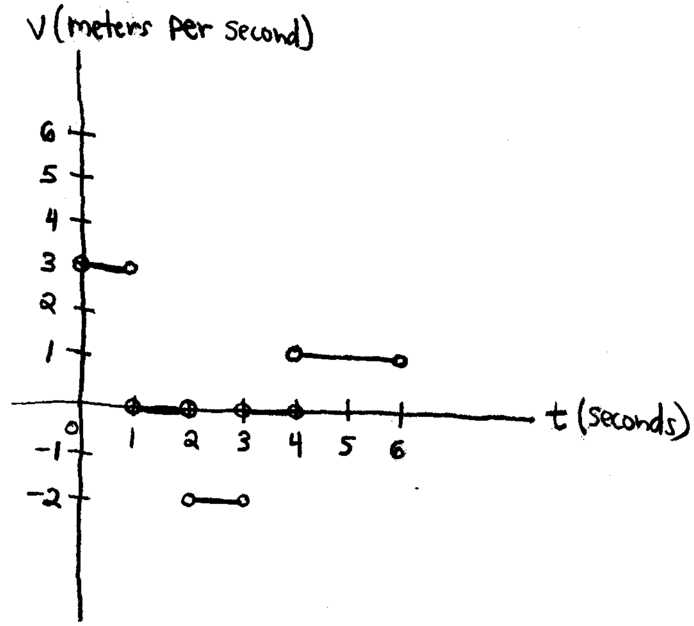
What does it mean for $g(x)$ to be continuous at $x = 5$? Use the limit definition of continuity to answer this question. (2 points)

$$\lim_{x \to 5} g(x) = g(5)$$

8. A particle starts by moving to the right along a horizontal line; the graph of its position function is shown below.



a) Draw a numerically accurate velocity versus time graph for the particle. (4 points)



b) During what time interval is the particle moving to the left? (1 point)

$2 < t < 3$

c) During what time intervals is the particle at rest? (1 point)

$1 < t < 2$ AND $3 < t < 4$