Quiz 8

128 Let C = total cost of constructing the box.
b = area of the base
s = area of one side

$$C = \underbrace{4}_{\$4 \text{ per sq. meter}} \overbrace{(2b)}^{\text{two bases}} + \underbrace{2}_{\$2 \text{ per sq. meter}} \overbrace{(4s)}^{\text{four sides}}$$

Note:

$b = w^2$

$s = wh$

$$C = 8b + 8s$$

$$C = 8(b + s)$$

$$C = 8\left[w^2 + wh\right]$$

Volume $= w^2 h$

given $\rightarrow 20 = w^2 h$

$\frac{20}{w^2} = h$

$$C = 8\left[w^2 + w\left(\frac{20}{w^2}\right)\right]$$

$$\boxed{C(w) = C = 8\left[w^2 + \frac{20}{w}\right]}$$

a) when $V = 20$ and $w = 1/2$ then $h = \frac{20}{(1/2)^2} = \frac{20}{1/4} = 80$ meters

$$C = 8\left[(1/2)^2 + \frac{20}{1/2}\right] = \$322$$

b) when $V = 20$ and $w = 4$ then $h = \frac{20}{4^2} = 1.25$ meters

$$C = 8\left[4^2 + \frac{20}{4}\right] = \$168$$

(129) $C(w)$ approaches infinity as $w \to \infty$ and as $w \to 0^+$

This means the box would cost a great deal if you made it extremely wide or extremely narrow.

TI-83+ key strokes for finding the minimum → $\boxed{2^{nd}}$ / $\boxed{\text{TRACE}}$ $\boxed{3}$ yield $C = \$111.40$ when $w = 2.1544$

(130) $C = 8\left[w^2 + \frac{20}{w}\right] = 8(w^2 + 20w^{-1})$

$$C' = 8(2w - 20w^{-2}) = 8\left[2w - \frac{20}{w^2}\right] = 0$$

$$2w - \frac{20}{w^2} = 0$$

$$w - \frac{10}{w^2} = 0$$

$$w = \frac{10}{w^2}$$

$$w^3 = 10$$

the width that costs the least → $w = 10^{1/3}$

$$C(10^{1/3}) = 8\left[(10^{1/3})^2 + \frac{20}{10^{1/3}}\right] = 8\left[10^{2/3} + 2 \cdot 10^{2/3}\right]$$

$h = \frac{20}{w^2} = \frac{20}{10^{2/3}} = 2 \cdot 10^{1/3}$

↑ the height that costs the least

$= 24 \cdot 10^{2/3}$ ← The exact least cost

$\approx \$111.398132006707$