Quiz 10

234) Let x be the time at which the balloon breaks during inflation. We are given that $V'(t) = 3\sqrt{t}$ where $V(t)$ is the amount of air in the balloon at time t. We may assume that $V(0) = 0$ since the balloon is completely deflated at $t = 0$. The maximum capacity of the balloon is $17 ft^3$ as given in the problem. Since this number represents the capacity of the balloon at the moment it breaks, we must solve $V(x) = 17$.

Now $V'(t) = 3\sqrt{t} = 3t^{1/2}$

so $V(t) = \int V'(t)\,dt = \dfrac{3t^{1/2+1}}{1/2+1} = \dfrac{3t^{3/2}}{3/2} = 2t^{3/2} + C$.

but $C = 0$ since $V(0) = 0$.

so $V(x) = 2x^{3/2}$.

Hence we solve $2x^{3/2} = 17$ and get $x = \left(\frac{17}{2}\right)^{2/3} \approx \boxed{4.165 \text{ minutes}}$

258 a) $f(x) = 10x^4 - 20x^3 + 10x^2 + x$

$f'(x) = 40x^3 - 60x^2 + 20x + 1$

$f'(0) = 1$ and $f(0) = 0$

$f'(1) = 40 - 60 + 20 + 1 = 0 + 1 = 1$ and $f(1) = 10 - 20 + 10 + 1$
$= 1$

Tangent line at $x=0$ is $y - 0 = 1(x-0)$ or $y = x$

Tangent line at $x=1$ is $y - 1 = 1(x-1)$ or $y = x$

same

b) $$\text{Area} = \int_0^1 (y_T - y_B)\,dx = \int_0^1 \left[(10x^4 - 20x^3 + 10x^2 + x) - (x)\right] dx$$

$$= \int_0^1 (10x^4 - 20x^3 + 10x^2)\,dx$$

$$= \left[2x^5 - 5x^4 + \frac{10}{3}x^3\right]_0^1$$

$$= (2 - 5 + 10/3) - (0 - 0 + 0)$$

$$= \boxed{\frac{1}{3}}$$