#270) Find out where the two curves intersect by solving $x^{5/2} = k^2\sqrt{x}$.

This implies $x^5 = k^4 x$

$$x^5 - k^4x = 0$$
$$x(x^4 - k^4) = 0$$
$$x(x^2-k^2)(x^2+k^2) = 0$$
$$x(x+k)(x-k)(x^2+k^2) = 0$$

The only non-negative solutions are $x=0$ and $x=k$ since k is positive. These solutions are the limits of integration.

$$\text{Volume} = \int_0^k \pi\left(k^2\sqrt{x}\right)^2 - \pi\left(x^{5/2}\right)^2 dx$$

$$= \pi \int_0^k k^4x - x^5 dx$$

$$= \pi\left[\frac{k^4}{2}x^2 - \frac{1}{6}x^6 dx\right]_0^k = \pi\left[\left(\frac{k^6}{2} - \frac{k^6}{6}\right) - (0-0)\right]$$

$$= \pi\frac{k^6}{3} = 243\pi$$

$$\Rightarrow k^6 = 729$$

$$\Rightarrow \boxed{k=3}$$

#237) Displacement $= \int_0^6 125 - x^3\,dx = \left[125x - \frac{1}{4}x^4\right]_0^6 = 426 - 0$

The plane is 426 miles north of it's position at $t=0$.

#258) $f'(x) = 40x^3 - 60x^2 + 20x + 1$

$f'(0) = 1$
$f'(1) = 40 - 60 + 20 + 1 = 1$

Both $(0,0)$ and $(1,1)$ lie on the graph of f.

1^{st} tangent line is $y - 0 = 1\cdot(x-0)$ or simply $y = x$.

2^{nd} tangent line is $y - 1 = 1\cdot(x-1)$ or simply $y = x$.

Area bounded between curves is $\int_0^1 10x^4 - 20x^3 + 10x^2\,dx$

$$= \left[2x^5 - 5x^4 + \frac{10}{3}x\right]_0^1 = \left(2 - 5 + \frac{10}{3}\right) - (0)$$

$$= \boxed{\frac{1}{3}}$$

#274) $y=x^2 \Rightarrow x=\sqrt{y}$

$y=mx \Rightarrow x=\frac{y}{m}$

Find pts. of intersection by solving:

$$x^2=mx$$
$$x^2-mx=0$$
$$x(x-m)=0$$
$$x=0 \text{ and } x=m$$

$$\left.\begin{array}{l} x=0 \Rightarrow y=0^2=0 \\ x=m \Rightarrow y=m^2 \end{array}\right\} \text{limits of integration}$$

$$\text{Volume} = \int_0^{m^2} \pi(\sqrt{y})^2-\pi\left(\frac{y}{m}\right)^2 dy$$

$$= \pi\int_0^{m^2} y-\frac{y^2}{m^2}\,dy$$

$$= \pi\left[\frac{y^2}{2}-\frac{y^3}{3m^2}\right]_0^{m^2}$$

$$= \pi\left[\frac{m^4}{2}-\frac{m^4}{3}\right] = \boxed{\frac{\pi m^4}{6}}$$