HW #8 Solutions

Final Answer:

Abs. max is $(2,0)$

Abs. min. is $(2-\sqrt{5}, -2.118)$

local min is $(2-\sqrt{5}, -2.118)$

There is a local max. but occurs outside of the interval $[-1,2]$ so we don't include it.

① $f(x) = \dfrac{x-2}{x^2+1}$ on $[-1,2]$ ← Interval.

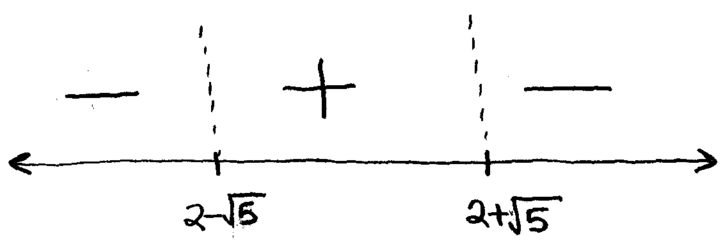
$$f'(x) = \frac{(x^2+1)(1)-(x-2)(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{x^2+1-2x^2+4x}{(x^2+1)^2} = \frac{-x^2+4x+1}{(x^2+1)^2}$$

We find the critical #'s by setting the numerator, in this case $-x^2+4x+1$, equal to 0. The denominator is never 0, so we don't need to worry about when the derivative is undefined. To this end, $-x^2+4x+1=0 \iff x^2-4x-1=0$. The quadratic formula gives us

$$x = \frac{4 \pm \sqrt{16-4(1)(-1)}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$ ← These are the critical #'s.

We put the critical #'s into a sign diagram and apply the 1st derivative test.



There is a local min. at $x=2-\sqrt{5}$ since there is a sign change in the derivative from neg. to pos. there.

There is a local max. at $x=2+\sqrt{5}$ since there is a sign change in the derivative from pos. to neg. there.

We now use the closed interval method to find the absolute extreme values of f. The endpts. are $x=-1$ and $x=2$ so

$f(-1) = -1.5$

largest → $f(2) = 0$

smallest → $f(2-\sqrt{5}) = -2.118$

exclude from list → $f(2+\sqrt{5}) = 0.118$

The abs. min is -2.118 which occurs at $x=2-\sqrt{5}$.

The abs. max is 0 which occurs at $x=2$.

Note: $2+\sqrt{5}$ is not in the interval so we exclude it from the list

(2) $I'(t) = 0.00045225t^4 + 0.005752t^3 - 0.19683t^2 + 0.9196t - 0.6270 = 0$

Critical #'s are:

$x_1 = 0.8231216212$ ← yields a local min.

$x_2 = 5.130907205$ ← yields a local max.

$x_3 = 11.04593628$ ← yields a local min.

↑
since $x_3 > 10$ it follows that x_3 lies outside the interval of interest namely $[0, 10]$

so we exclude it from the problem

The closed interval method gives us:

$f(x_1) = 99.08945267$

$f(x_2) = 100.6730555$ ← largest (abs. max)

$f(0) = 99.33$

$f(10) = 96.854$ ← smallest (abs. max)

$1984 + 10$ is 1994 (when food was cheapest)

$1984 + x_2$ is 1989.130907 (when food was most expensive)