HW #6

① $f(x) = (x^3+1)(5-x) - 5$ = the displacement function (time is x here)

$f(0) = 0$ $\qquad f(2) = 22$

$f(1) = 3$ $\qquad f(4.95) = 1.11436875$

② $f'(x) = (3x^2)(5-x) + (x^3+1)(-1) + 0$ = velocity function

$= 15x^2 - 3x^3 - x^3 - 1$

$= -4x^3 + 15x^2 - 1$

$f'(0) = -1$ $\qquad f'(2) = 27$

$f'(1) = 10$ $\qquad f'(4.95) = -118.612$

③ Solve $-4x^3+15x^2-1=0$ using $\boxed{2^{nd}}$ $\boxed{TRACE}$ intersect to obtain:

$x = 3.732050808$

$x = 0.2679491924$

$x = -0.25 \leftarrow$ Since x represents time and time is never negative we eliminate this possibility.

Also $-4x^3+15x^2-1=0$ if and only if $4x^3-15x^2+1=0$. This can be factored into:

$$(4x+1)(x^2-4x+1)=0$$

This factorization works because of the rational roots thm and synthetic division.

i.e:

$$\begin{array}{r|rrrr} & 4 & -15 & 0 & 1 \\ -\frac{1}{4} & & -1 & 4 & -1 \\ \hline & 4 & -16 & 4 & \boxed{0} \end{array} \leftarrow \text{remainder} \quad \Rightarrow \left(x+\tfrac{1}{4}\right)\left(4x^2-16x+4\right)=0$$

Now factor out a 4 in the 2^{nd} factor and mult. the 1^{st} factor by 4 to get $(4x+1)(x^2-4x+1)=0$

(OVER)

Set each factor equal to zero to obtain: $4x+1=0$ and $x^2-4x+1=0$

The first factors yeilds $x=\frac{-1}{4}$ while the second yeilds $x=2\pm\sqrt{3}$ by the quadratic formula

These numbers are the <u>exact</u> answers. Notice that $2+\sqrt{3}\approx 3.732050808$

and $2-\sqrt{3}\approx 0.2679491924$

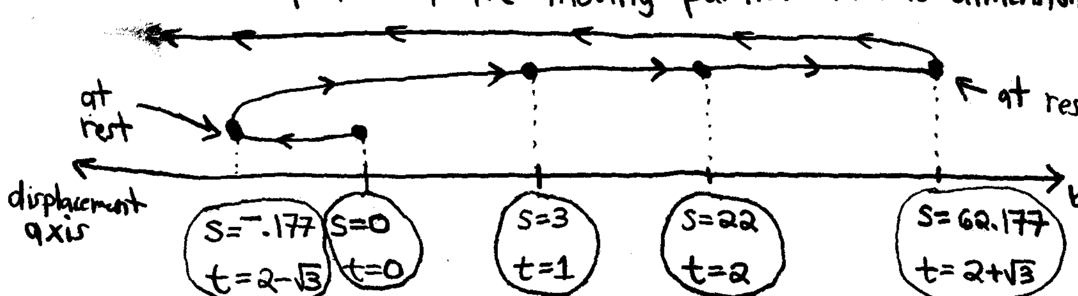
and $\frac{-1}{4}=-0.25$

Thus the partical is at rest when $x=2\pm\sqrt{3}$ since we eliminate $\frac{-1}{4}$ as time is never negative.

The particle is moving forward when $f'(x)=-4x^3+15x^2-1>0$ this happens when

$2-\sqrt{3}<x<2+\sqrt{3}$ as seen from the graph of $y=f'(x)$ on the graphing calc.

④ Here is a picture of the moving particle in one dimension:



example plug in $x=2+\sqrt{x}$ into $f(x)$ to get 62.177

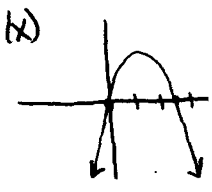
we mult. by 2 here because the particle turns around during the first second.

The particle traveled $(0.1769145362)\cdot 2+3=\boxed{3.353829072\text{ ft.}}$ during the 1^{st} Second.

The particle traveled $22-3=\boxed{19\text{ ft.}}$ between seconds 1 and 2 because the particle had a higher average velocity between seconds 1 and 2 than it did during the first sec. thus it was able to cover a greater distance in the same amount of time.

⑤ $f''(x)=-12x^2+30x=$ the acceleration function.

$y=f''(x)$



Note: $-12x^2+30x=0$

$2x(-6x+15)=0$ so $x=0$ or $x=\frac{5}{2}$

HW #6 continued...

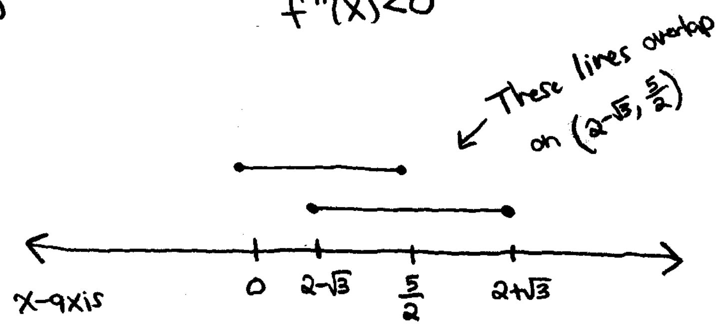
⑥

The particle speeds up when (I) $f'(x)>0$ and $f''(x)>0$ **OR** when (II) $f'(x)<0$ and $f''(x)<0$

I

$f'(x)>0$ when $2-\sqrt{3}<x<2+\sqrt{3}$

$f''(x)>0$ when $0<x<\frac{5}{2}$

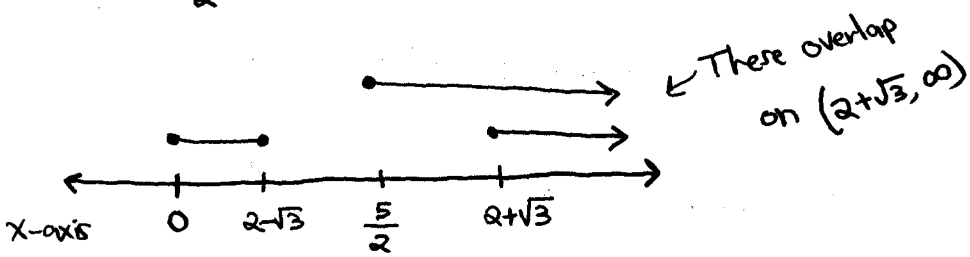


★ The particle speeds up on the interval(s) where the two line segments overlap. ★

II

$f'(x)<0$ when $0<x<2-\sqrt{3}$ **OR** $x>2+\sqrt{3}$

$f''(x)<0$ only when $x>\frac{5}{2}$ since time is never negative.



The particle is speeding up on $(2-\sqrt{3}, \frac{5}{2}) \cup (2+\sqrt{3}, \infty)$

The particle is slowing down when it is not speeding up nor at rest. This happens on the interval $(0, 2-\sqrt{3}) \cup (\frac{5}{2}, 2+\sqrt{3})$.