GROUP WORK 1, SECTION 3.3
Doing a Lot with a Little

Section 3.3 introduces the Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$, where n is any real number. The good news is that this rule, combined with the Constant Multiple and Sum Rules, allows us to take the derivative of even the most formidable polynomial with ease! To demonstrate this power, try Problem 1:

1. *A formidable polynomial:*

$$f(x) = x^{10} + \tfrac{7}{9}x^9 + \tfrac{1}{2}x^8 - 5x^7 - 0.33x^6 + \pi x^5 - \sqrt{2}x^4 - 42$$

Its derivative:

$$f'(x) =$$

The ability to differentiate polynomials is only one of the things we've gained by establishing the Power Rule. Using some basic definitions, and a touch of algebra, there are all kinds of functions that can be differentiated using the Power Rule.

2. *All kinds of functions:*

$$f(x) = \sqrt[3]{x} + \sqrt[5]{2} \qquad g(x) = \frac{1}{x^3} - \frac{1}{\sqrt[4]{x^3}} \qquad h(x) = \frac{x^5 - 3\sqrt{x} + 2}{\sqrt{x}}$$

Their derivatives:

$$f'(x) = \qquad g'(x) = \qquad h'(x) =$$

Unfortunately, there are some deceptive functions that look like they should be straightforward applications of the Power and Constant Multiple Rules, but actually require a little thought.

3. *Some deceptive functions:*

$$f(x) = (2x)^4 \qquad g(x) = \left(x^3\right)^5$$

Their derivatives:

$$f'(x) = \qquad g'(x) =$$

The process you used to take the derivative of the functions in Problem 3 can be generalized. In the first case, $f(x) = (2x)^4$, we had a function that was of the form $(kx)^n$, where k and n were constants ($k = 2$ and $n = 4$). In the second case, $g(x) = \left(x^3\right)^5$, we had a function of the form $\left(x^k\right)^n$. Now we are going to find a pattern, similar to the Power Rule, that will allow us to find the derivatives of these functions as well.

4. Show that your answers to Problem 3 can also be written in this form:

$$f'(x) = 4(2x)^3 \cdot 2 \qquad g'(x) = 5\left(x^3\right)^4 \cdot 3x^2$$

And now it is time to generalize the Power Rule. Consider the two general functions, and try to find expressions for the derivatives similar in form to those given in Problem 4. You may assume that n is an integer.

5. *Two general functions*:

$$f(x) = (kx)^n \qquad g(x) = \left(x^k\right)^n$$

Their derivatives:

$$f'(x) =$$
$$g'(x) =$$