HW 9 Solutions

① $x^{\frac{1}{3}} \cdot (x-4) = 0$

$x^{\frac{1}{3}} = 0$ and $x - 4 = 0$

$\boxed{x = 0 \quad \text{and} \quad x = 4}$ are the roots of f.

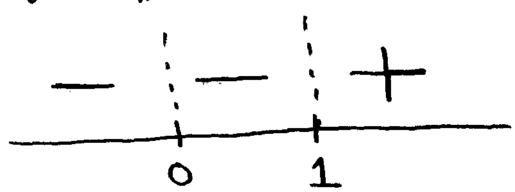
② $f'(x) = \frac{1}{3} x^{-2/3} \cdot (x-4) + x^{1/3}(1)$

$$= \frac{x-4}{3x^{2/3}} + \frac{x^{1/3}}{1} = \frac{x-4+3x}{3x^{2/3}} = \frac{4x-4}{3x^{2/3}} = \frac{4(x-1)}{3x^{2/3}}$$

set $4(x-1) = 0$ AND $3x^{2/3} = 0$

$\boxed{x = 1 \quad \text{and} \quad x = 0}$ are the critical #'s of f.

③ Sign chart



Increase on $(1, \infty)$

Decrease on $(-\infty, 1)$

④ The 1st derivative changes sign from negative to positive at $x = 1$ and thus there is a local minimum at $x = 1$. There are NO local maximums.
$y = -3$

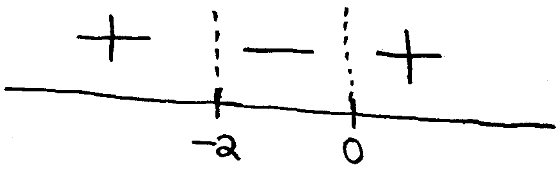
⑤ $$f''(x) = \frac{12x^{2/3} - 4(x-1) \cdot 2x^{-\frac{1}{3}}}{9x^{4/3}} = \frac{\frac{12x^{2/3}}{1} - \frac{8(x-1)}{x^{1/3}}}{\frac{9x^{4/3}}{1}} = \frac{12x - 8(x-1)}{x^{1/3}} \cdot \frac{1}{9x^{4/3}}$$

$$= \frac{4x+8}{9x^{5/3}} = 0$$

set $4x + 8 = 0$ and $9x^{5/3} = 0$

$\boxed{x = -2 \qquad x = 0}$ are the only critical #'s of f'.

⑥ Sign chart



f is concave up on $(-\infty, 2) \cup (0, \infty)$

f is concave down on $(-2, 0)$

⑦ $(-2, 7.56)$ and $(0,0)$ are the only two inflection pts. of f.
↑
an approximation, since f changes concavity at $x=-2$ and $x=0$ only.

⑧

$f(x) = \sqrt[3]{x}\,(x-4)$