HW #11

1) The maximum value for x in this problem with an $8\frac{1}{2}''$ by $11''$ sheet of paper is half the shortest side, which in this case, is $4.25''$

$$V(x) = x(8.5-2x)(11-2x)+x^3 \quad \text{on } [0, 4.25] \leftarrow \text{restricted domain}$$

$$V(x) = 5x^3 - 39x^2 + 93.5x$$

1st derivative test

$$V'(x) = 15x^2 - 78x + 93.5 = 0$$

+		−		+
	x_1		x_2	

critical #'s are $x_1 = 1.874281965$
$x_2 = 3.325718035$

since $V'(x)$ changes sign from $(+)$ to $(-)$ at x_1 this critical # yields a local max. However we use the closed interval method to solve this problem.

x	$V(x)$
0	0
x_1	71.162115
x_2	63.517885
4.25	76.765625 ← max.

The absolute max. volume is 76.766 in^3 which occurs at the end pt. 4.25.

2) The open topped box that results in the maximal case is a strip of paper with dimensions $2.5''$ by $8.5''$. (Zero height) This happens because the max. occured at an endpt. leaving no room for the height in the open-topped box.

3) $V(x) = x(6-2x)(10-2x)+x^3$ on $[0, 3]$
$V(x) = 5x^3 - 32x^2 + 60x$

$V'(x) = 15x^2 - 64x + 60 = 0$
critical #'s are $x_1 = 1.390964752$
$x_2 = 2.875701915$

x	$V(x)$
0	0
x_1	35.000905
x_2	26.818354
3	27

The abs. max. volume is 35.000905 in^3 which occurs at x_1