MAC 2311 CALCULUS WITH ANALYTIC GEOMETRY I
TEST 1

Name ___KEY___   Score ___53 pts.___ (out of)

Directions: Answer each question showing <u>ALL</u> work for full credit.

1. Use the graph of the function below to determine each statement as TRUE or FALSE. No work is required for this particular problem. (8 points)
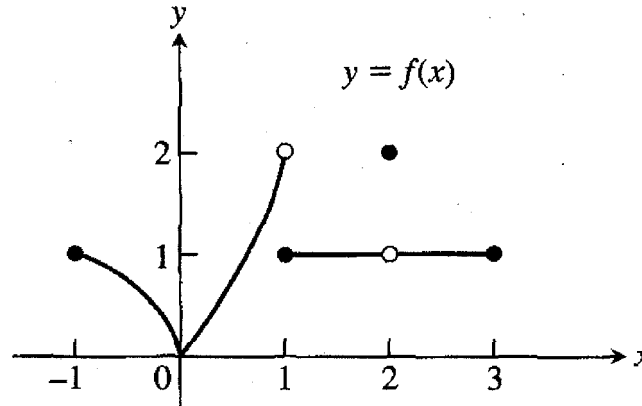
___T___ $\lim\limits_{x \to -1^+} f(x) = 1$

___F___ $\lim\limits_{x \to 2} f(x) = 2$

___F___ $\lim\limits_{x \to a} f(x)$ exists for all numbers a in the interval $(-1,3)$ except for 1 and 2.

___F___ $\lim\limits_{x \to 1^-} f(x) = f(1)$

___T___ $\lim\limits_{x \to 1} f(x)$ does not exist

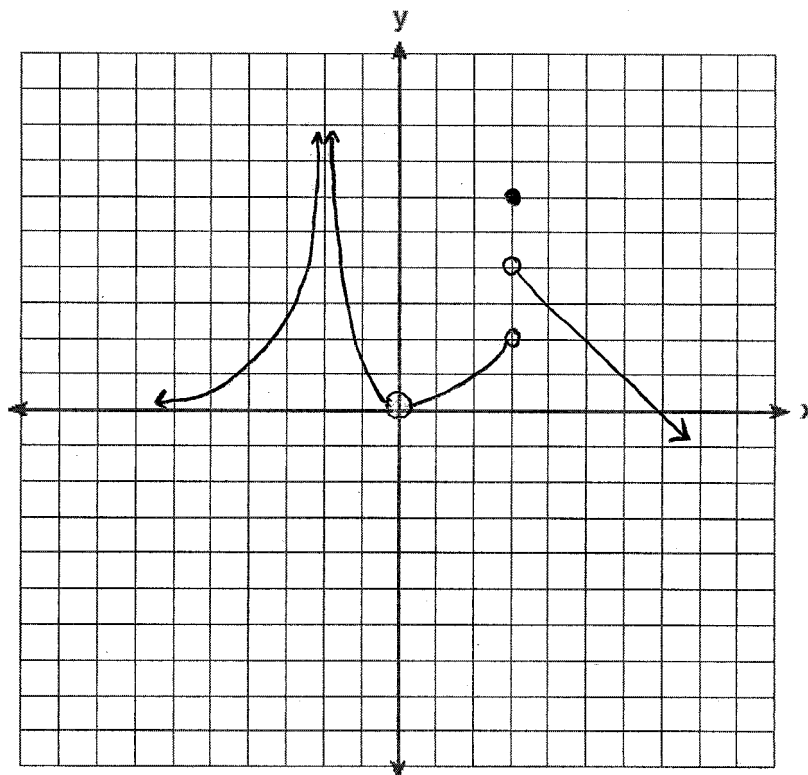___T___ $f(x)$ is continuous at $x = \dfrac{3}{2}$

___T___ $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^-} f(x)$

___F___ $f(x)$ is continuous from the right at $x = 2$

2. Sketch the graph of a function that satisfies all of the conditions below. (8 points)

- $\lim\limits_{x \to 3^+} f(x) = 4$

- $\lim\limits_{x \to 3^-} f(x) = 2$

- $\lim\limits_{x \to -2} f(x) = +\infty$

- $\lim\limits_{x \to 0} f(x) = 0$

- $f(3) = 6$

- $f(0)$ is undefined

- $f(x) = -x + 7$ for $x > 3$

Note: This is ONE Possibility.

3. The definition for the derivative of a function f is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Part a) Use this definition to find the derivative of the function $f(x) = -\dfrac{1}{x}$ (4 points)

$$f'(x) = \lim_{h \to 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)} = \lim_{h \to 0} \frac{-x+x+h}{x \cdot h(x+h)}$$

$$= \lim_{h \to 0} \frac{1}{x(x+h)}$$

$$\overset{D.S.P}{=} \frac{1}{x^2}$$

Part b) You are given that $f'(0.5) = 4$. Use this information to find the equation of the tangent line to the graph of f when $x = 0.5$ (2 points)

$$f(0.5) = \frac{-1}{0.5} = -2 = y_1$$

$$y + 2 = 4\left(x - \tfrac{1}{2}\right)$$

or

$$\boxed{y = 4x - 4}$$

4. A cardiac monitor is used to compile the number of heartbeats for a patient. The values in the table below show the cumulative number of heartbeats H after t minutes. Here H is a function of time. For example $H(38) = 2661$.

P   Q

t (min.)	36	38	40	42	44	46
H	2530	2661	2806	2948	3080	3202

a) If P is the point $(42, 2948)$ on the graph of H, find the slope of the secant line PQ where Q is the point on the graph when $t = 46$. (1 points)

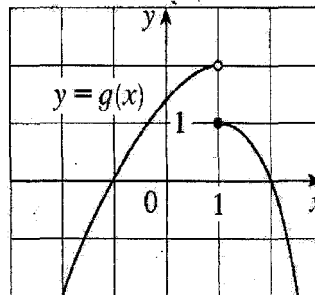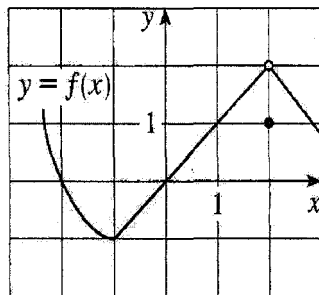$$m_{sec} = \frac{3202 - 2948}{46 - 42} = \frac{254}{4} = 63.5$$

b) Give the <u>best</u> possible estimate for the slope of the tangent line at P by averaging the slopes of two secant lines. (2 points)

$$m_{tan} \approx \frac{\frac{2948 - 2806}{42 - 40} + \frac{3080 - 2948}{44 - 42}}{2} = \frac{71 + 66}{2} = 68.5$$

c) Write a sentence or two, using layman's terms, describing what the equation $H'(42) = 68$ means in context of this problem. Be specific and including units of measurement. (2 points)

42 minutes after being hooked up to a cardiac monitor, the patient had a heartrate of 68 beats per minute.

5. The graphs of f and g are given below. Use them to evaluate each limit, if it exists. If the limit does not exist write DNE in the space provided. (6 points)

$\displaystyle\lim_{x \to 2}[f(x) + g(x)] = \underline{\quad 2 \quad}$ $\displaystyle\lim_{x \to 1}[f(x) + g(x)] = \underline{\text{DNE}}$ $\displaystyle\lim_{x \to 0}[f(x)g(x)] = \underline{\quad 0 \quad}$

$\displaystyle\lim_{x \to -1}\frac{f(x)}{g(x)} = \underline{\text{DNE}}$ $\displaystyle\lim_{x \to 2}x^3 f(x) = \underline{\quad 16 \quad}$ $\displaystyle\lim_{x \to 1}\sqrt{3 + f(x)} = \underline{\quad 2 \quad}$

or

$\pm\infty$

6. Use algebra to compute both of the following limits, if they exist. (8 points)

$$\lim_{x \to -1} \left(\frac{x^2 + 3x + 2}{x^2 - x - 2} \right)$$

$$= \lim_{x \to -1} \frac{(x+1)(x+2)}{(x+1)(x-2)}$$

$$= \lim_{x \to -1} \frac{x+2}{x-2}$$

$$\underset{=}{DSP} \boxed{\frac{-1}{3}}$$

$$\lim_{x \to 0} \left(\frac{\sqrt{1+x} - 1}{x} \right) \cdot \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$$

$$= \lim_{x \to 0} \frac{1 + x - 1}{x \left[\sqrt{1+x} + 1 \right]}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{1+x} + 1}$$

$$\underset{=}{DSP} \boxed{\frac{1}{2}}$$

7. Use one-sided limits to find the constant c that makes g continuous everywhere. (4 points)

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

$$\lim_{x \to 4^-} g(x) = \lim_{x \to 4^-} (x^2 - c^2) \overset{\text{D.S.P}}{=} 4^2 - c^2 = 16 - c^2$$

$$\lim_{x \to 4^+} g(x) = \lim_{x \to 4^+} (cx + 20) \overset{\text{D.S.P.}}{=} 4c + 20 = g(4)$$

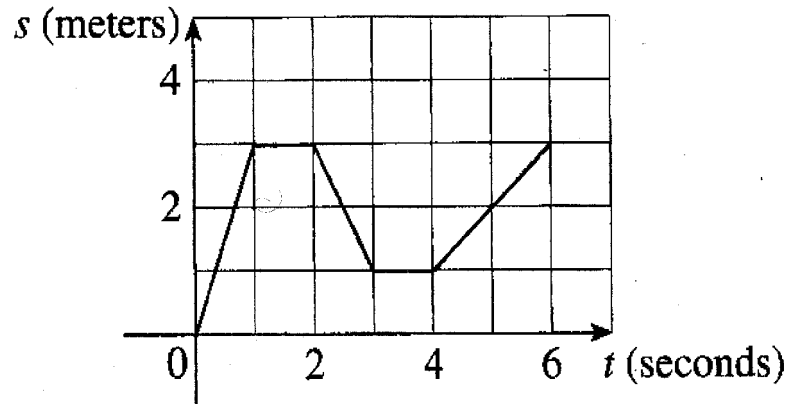solve $16 - c^2 = 4c + 20$

$$c^2 + 4c + 4 = 0$$

$$(c + 2)^2 = 0$$

$$\boxed{c = -2}$$

What does it mean for the function g to be continuous at $x = 5$? Use the limit definition of continuity to answer this question. (2 points)

$$\lim_{x \to 5} g(x) = g(5) = 5c + 20 = 10$$

8. A particle starts by moving to the right along a horizontal line; the graph of its position function is shown below.

a) Draw a numerically accurate velocity versus time graph for the particle. (4 points)

all end pts. are open.
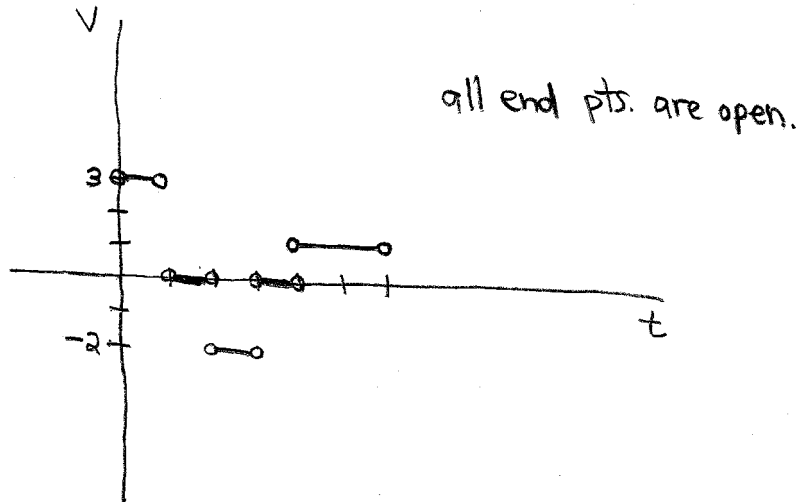
b) During what time interval is the particle moving the fastest? (1 point)

$$0 < t < 1$$

c) During what time interval(s) is the particle at rest i.e. not moving? (1 point)

$$1 < t < 2 \quad \text{AND} \quad 3 < t < 4$$