MAC 2233 CALCULUS FOR BUSINESS AND SOCIAL SCIENCE
TEST 2

$\overline{\quad 48 \quad}$

Name KEY Score

Part I - Multiple Choice. Choose the answer that best answers each question. (3 points each)

1 Find all the critical numbers of the function $f(x) = x^3 - 48x - 6$

A) None
B) $-4, 4$
C) $0, -4, 4$
D) $\sqrt[3]{-6}$

2 An equation for the tangent line to the curve $y = (x^7 + x - 1)^5$ at the point where $x = 1$ is

A) $y = 40x - 39$
B) $y = 40x$
C) $y = 7x + 1$
D) $y = 40x - 1$

3 Find the absolute minimum of the function $f(x) = x^3 - 3x^2$ on the interval $-1 \le x \le 3$.

A) -1
B) -4
C) 0
D) 3

4 Determine where the graph of $f(x) = x^3 - 3x^2 - 9x + 1$ is concave down.

A) $x > 1$
B) $x < 1$
C) $x > -1$
D) $x < -1$

5 Locate all inflection points of $f(x) = x^4 + 6x^3 - 24x^2 + 26$.

A) (1, 9) and (–4, –486)
B) (1, 9)
C) None
D) (0, 26)

6 Find the intervals of increase and decrease for the function $f(x)=x^2+5x-3$.

A) Decreasing for $x<-\frac{5}{2}$; increasing for $x>-\frac{5}{2}$

B) Decreasing for $x>-\frac{5}{2}$; increasing for $x<-\frac{5}{2}$

C) Decreasing for all x

D) Increasing for all x

7 Find $\frac{dy}{dx}$, where $xy^3-3x^2=7y$.

A) y^3-6x-7

B) $\frac{6x-y^3}{3xy^2-7}$

C) y^3-6x

D) $\frac{6x^2}{y^3}$

8 The cost of producing x units of a certain commodity is $C(x)=3x^2+3x+4$ dollars. If the price is $p(x) = (43 - x)$ dollars per unit, determine the level of production that maximizes profit.

A) $x=1$

B) $x=2$

C) $x=3$

D) $x=5$

9 The demand function for a certain commodity is $D(p)=\frac{30}{p+5}$. For what values of p is the demand inelastic?

A) $p>0$

B) $p<0$

C) $p>-5$

D) $p<-5$

Part II – Short Answer. Answer each question showing ALL work for full credit.

10. Suppose the total cost in dollars of manufacturing x units is $C(x) = 3x^2 + x + 500$.

a. Use marginal analysis to estimate the cost of manufacturing the 41[th] unit. (2 points)

$C'(x) = 6x + 1$

use $C'(40)\Delta x$ where $\Delta x = 1$

so $C'(40) = \$241$

b. Compute the actual cost of manufacturing the 41[st] unit. (2 points)

$C(41) - C(40) = 5584 - 5340$

$= \$244$

c. Find the average cost per unit when the production level is 40 units. (1 point)

$A(x) = \frac{3x^2 + x + 500}{x} = 3x + 1 + \frac{500}{x}$

$A(40) = \$133.50$ per unit

11. A company determines that the consumer demand curve for tissues is given by $q = 2500 - 50p$ where $0 \le p \le 50$ and p is the price, in dollars, per case of tissues it charges to consumers while q is the quantity demanded. The tissues cost \$30 per case to produce. Assume that the fixed costs to produce the tissues is \$0.

a. Find the price the tissues should be sold for in order to maximize the profit. (3 points)

$$C = 30q = 30(2500-50p) = 75000 - 1500p$$

$$R = p \cdot q = (2500-50p)p = 2500p - 50p^2$$

$$P = R - C = -50p^2 + 2500p - 75000 + 1500p$$

$$P = -50p^2 + 4000p - 75000$$

$$P' = -100p + 4000 = 0$$

$$p = \$40$$

$+ \;|\; -$

40

max

b. What is the maximum profit? (1 point)

plug in 40 get \$ 5000

c. What is the total revenue when the demand is 200 cases of tissues? (1 point)

$$200 = 2500 - 50p$$

$p = 46$ $\qquad R(46) = 2500(46) - 50(46)^2 = \$9200$

d. Compute the marginal revenue in terms of p. (1 point)

$$\frac{dR}{dp} = 2500 - 100p$$

12. A certain company has determined that the average cost per unit for manufacturing x units of their most popular product is given by $A(x) = x + \dfrac{64}{x} - 3$ for $0 \le x \le 50$.

a. Construct a sign chart that shows the intervals where $A(x)$ is increasing and decreasing. Remember that you are dealing with a restricted domain. (2 points)

$$A'(x) = 1 - \frac{64}{x^2} = 0$$

$$x^2 = 64$$

$$x = 8$$

dec. | Inc.

— | +

8

b. How many units must be produced in order to minimize the average cost per unit? (1 point)

8 units

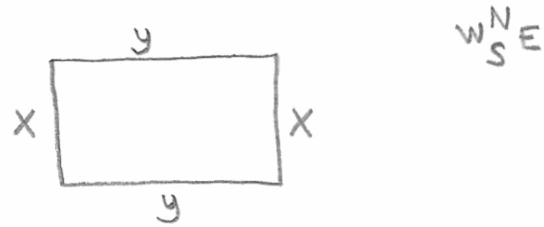
c. What is the minimum average cost per unit? (1 point)

$13

d. Compute the 2[nd] derivative of $A(x)$. (1 point)

$$A''(x) = \frac{128}{x^3}$$

13. You want to fence in a rectangular vegetable patch. The fencing for the east and west sides costs \$6 per foot, while the north and south sides costs only \$2 per foot. You have a budget of \$120 for the project. What is the largest area you can enclose? (5 points)

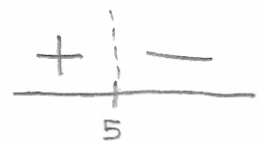


$P = 2x + 2y$

$C = 6(2x) + 2(2y) = 12x + 4y = 120 \rightarrow y = \dfrac{120 - 12x}{4} = 30 - 3x$

$A = x \cdot y$

$A = x(30 - 3x) = -3x^2 + 30x$



$A' = -6x + 30 = 0$

$x = 5$

$A_{max} = 75$ sq. ft.

A' changes sign from (+) to (−) at $x = 5$ so this yields a local max.