MAC 1105 COLLEGE ALGEBRA
TEST 3

Name KEY Score 40

Directions: Answer each question showing ALL work for full credit.

1. The formula for the volume of a cylinder is $V = \pi r^2 h$ where r is the radius and h is the height of the cylinder. A large cylindrical bottle of shampoo has a height of 20 centimeters. What radius should the bottle have if it must hold 769 cubic centimeters of shampoo? Use 3.14 as an approximation for π and round your answer to the nearest tenth. (3 points)

$$769 = (3.14)r^2(20)$$

$$r^2 = \frac{769}{(20)(3.14)}$$

$$r = \sqrt{\frac{769}{62.8}} = 3.5 \text{ cm}$$

2. Solve the following equation by using the **Zero-Product Property**. (Hint: first put it into standard form and then factor.) (4 points)

$$(2x-3)(x+1) = 3$$

$$2x^2+2x-3x-3=3$$

$$2x^2-x-6=0$$

$$(2x+3)(x-2)=0$$

$$2x+3=0 \text{ or } x-2=0$$

$$x=\frac{-3}{2} \text{ or } x=2$$

3. Solve the following equation using the **Quadratic Formula**. Write your answer with the square-root sign. DO NOT use decimals in your final answer. (4 points)

$$x(2x+5) = 8x+1$$

$$2x^2+5x-8x-1=0$$

$$2x^2-3x-1=0$$

$$a=2$$
$$b=-3$$
$$c=-1$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-1)}}{2(2)}$$

$$= \frac{3\pm\sqrt{17}}{4}$$

4. Solve the following equation by using an appropriate substitution and factoring. (4 points)

$$2x^{\frac{1}{2}} + 5 = 11x^{\frac{1}{4}}$$

$$2x^{\frac{1}{2}} - 11x^{\frac{1}{4}} + 5 = 0$$

Let $y = x^{\frac{1}{4}}$ then $y^2 = (x^{\frac{1}{4}})^2 = x^{\frac{2}{4}} = x^{\frac{1}{2}}$

$$2y^2-11y+5=0$$

$$(2y-1)(y-5)=0$$

$$y=\frac{1}{2} \text{ or } y=5$$

$$x^{\frac{1}{4}}=\frac{1}{2} \text{ or } x^{\frac{1}{4}}=5$$

$$x=\left(\frac{1}{2}\right)^4 \text{ or } x=5^4 \rightarrow x=\frac{1}{16} \text{ or } x=625$$

5. Part a) Write a quadratic equation whose solutions are $x = -4$ and $x = \frac{2}{3}$. Your equation should be in standard form with integer coefficients. (2 points)

$x+4=0$ and $3x-2=0$

$(x+4)(3x-2)=0$

$3x^2-2x+12x-8=0$

$3x^2+10x-8=0$

Part b) Convert the equation $y = 3x^2 - 6x - 1$ into vertex form. (3 points)

$a=3$

$b=-6$

$c=-1$

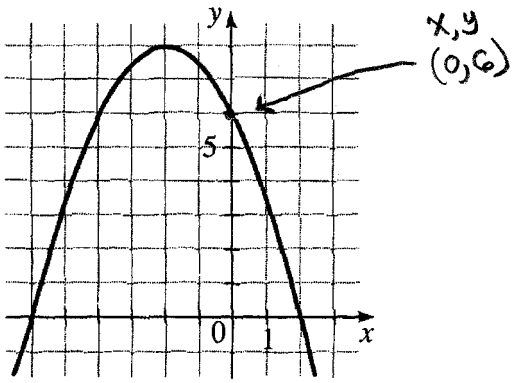
$y=a(x-h)^2+k$

$y=3(x-1)^2-4$

$h=\frac{-b}{2a}=\frac{-(-6)}{2(3)}=\frac{6}{6}=1$

$k=3(1)^2-6(1)-1=3-6-1=-4$

6. Part a) Use algebra to find the equation for the parabola whose graph is shown below. Put your equation into standard form. Be sure to count the squares correctly. (5 points)



$x=-6$ or $x=2$ are the x-intercepts

$y=a(x+6)(x-2)$

$6=a(6)(-2)$

$a=\frac{-1}{2}$

$y=\frac{-1}{2}(x+6)(x-2)$

$y=\frac{-1}{2}(x^2+4x-12)$

$y=\frac{-1}{2}x^2-2x+6$

Part b) Is the discriminant positive, negative or zero for the parabola above? (2 points)

Positive There are two x-intercepts

7.

What are the dimensions of a rectangle that has a perimeter of 300 meters and an area of 1200 square meters? Set-up and solve a quadratic equation to answer this question. Round your answer to the nearest tenth of a meter. (6 points)

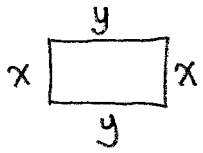
If you use your calculator to solve the equation then write down a window that allows you to see all of the following parts of the graph for full credit:

- The vertex
- The $y-$intercept
- Both $x-$intercepts

Window

$x-\min:\ -50$ $\quad y-\min:\ -5000$

$x-\max:\ 200$ $\quad y-\max:\ 5000$



$P = 2x+2y = 300 \rightarrow x+y = 150 \rightarrow y = 150-x$

$A = x\cdot y = 1200 \rightarrow x(150-x) = 1200$

$150x - x^2 = 1200$

$x^2 - 150x + 1200 = 0$

$a = 1$

$b = -150$

$c = 1200$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{150 \pm \sqrt{17700}}{2} = 75 \pm 5\sqrt{177}$$

$x = 8.48$ or $x = 141.52$

$y = 141.52$ $\quad y = 8.48$ $\quad$ so 8.5 m by 141.5 m

8. A toy rocket is fired into the air from ground level. The height above ground level of the rocket in feet is given by the equation $h = 96t - 16t^2$ where t is the time after launch in seconds.

a) Find the maximum height of the toy rocket. (2 points)

$t = \frac{-b}{2a} = \frac{-96}{2(-16)} = 3$ seconds

$h = 96(3) - 16(3)^2 =$ 144 feet

b) How long will it take the toy rocket to hit the ground after launch? (2 points)

Solve $-16t^2 + 96t = 0$

$-16t(t-6) = 0$

$t = 0$ or $t = 6$ seconds

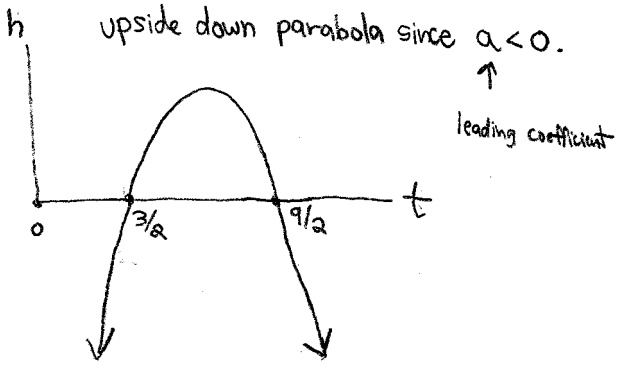
c) During what time intervals does the toy rocket have a height of less than 108 feet? (3 points)

Solve $-16t^2 + 96t < 108$

$-16t^2 + 96t - 108 < 0$

$-4(4t^2 - 24t + 27) < 0$

$-4(2t-3)(2t-9) < 0$



$[0, 3/2) \cup (9/2, 6]$ or $t < \frac{3}{2}$ and $t > \frac{9}{2}$

$t < 1.5$ and $t > 4.5$