1.

$$A=\int_{x=0}^{x=4}\left(y_T-y_B\right)dx=\int_0^4\left[\left(5x-x^2\right)-x\right]dx=\int_0^4\left(4x-x^2\right)dx$$

$$=\left[2x^2-\frac{1}{3}x^3\right]_0^4=\left(32-\frac{64}{3}\right)-(0)=\frac{32}{3}$$

2. $A=\int_0^6\left[2x-\left(x^2-4x\right)\right]dx=\int_0^6\left(6x-x^2\right)dx=\left[3x^2-\frac{1}{3}x^3\right]_0^6=108-72=36$

3.

$$A=\int_{y=0}^{y=1}\left(x_R-x_L\right)dy=\int_0^1\left[\sqrt{y}-(y^2-1)\right]dy=\int_0^1\left(y^{1/2}-y^2+1\right)dy$$

$$=\left[\frac{2}{3}y^{3/2}-\frac{1}{3}y^3+y\right]_0^1=\left(\frac{2}{3}-\frac{1}{3}+1\right)-(0)=\frac{4}{3}$$

4.

$$A=\int_0^3\left[\left(2y-y^2\right)-\left(y^2-4y\right)\right]dy=\int_0^3\left(-2y^2+6y\right)dy$$

$$=\left[-\frac{2}{3}y^3+3y^2\right]_0^3=(-18+27)-0=9$$

5.

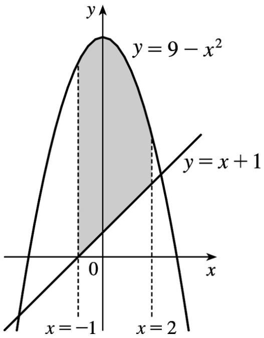
$$A=\int_{-1}^{2}\left[\left(9-x^2\right)-(x+1)\right]dx$$

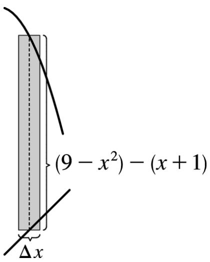
$$=\int_{-1}^{2}\left(8-x-x^2\right)dx$$

$$=\left[8x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{2}$$

$$=\left(16-2-\frac{8}{3}\right)-\left(-8-\frac{1}{2}+\frac{1}{3}\right)$$

$$=22-3+\frac{1}{2}=\frac{39}{2}$$





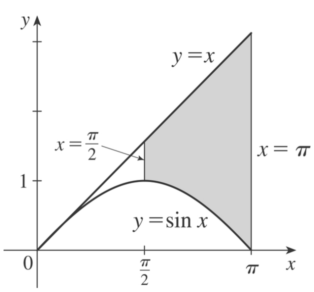
6.

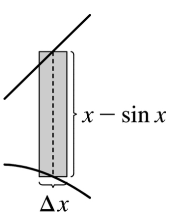
$$A = \int_{\pi/2}^{\pi} (x - \sin x)\, dx$$

$$= \left[\frac{x^2}{2} + \cos x \right]_{\pi/2}^{\pi}$$

$$= \left(\frac{\pi^2}{2} - 1 \right) - \left(\frac{\pi^2}{8} + 0 \right)$$

$$= \frac{3\pi^2}{8} - 1$$

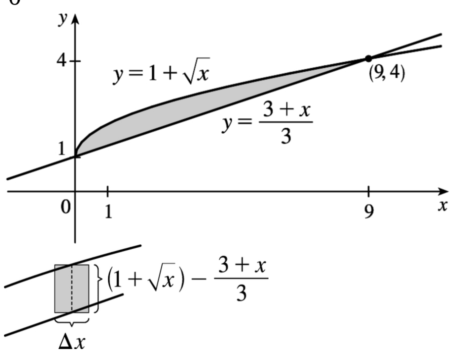




10. $1+\sqrt{x}=\dfrac{3+x}{3}=1+\dfrac{x}{3}\Rightarrow\sqrt{x}=\dfrac{x}{3}\Rightarrow x=\dfrac{x^2}{9}\Rightarrow 9x-x^2=0\Rightarrow x(9-x)=0\Rightarrow x=0$ or 9 , so

$$A=\int_0^9\left[\left(1+\sqrt{x}\right)-\left(\frac{3+x}{3}\right)\right]dx=\int_0^9\left[\left(1+\sqrt{x}\right)-\left(1+\frac{x}{3}\right)\right]dx=$$

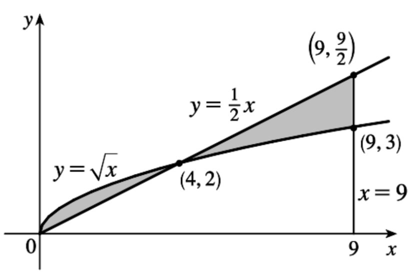
$$\int_0^9\left(\sqrt{x}-\frac{1}{3}x\right)dx=\left[\frac{2}{3}x^{3/2}-\frac{1}{6}x^2\right]_0^9=18-\frac{27}{2}=\frac{9}{2}$$

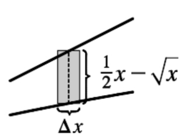


15. $\dfrac{1}{2}x=\sqrt{x}\Rightarrow\dfrac{1}{4}x^2=x\Rightarrow x^2-4x=0\Rightarrow x(x-4)=0\Rightarrow x=0$ or 4 , so

$$A=\int_0^4\left(\sqrt{x}-\frac{1}{2}x\right)dx+\int_4^9\left(\frac{1}{2}x-\sqrt{x}\right)dx=\left[\frac{2}{3}x^{3/2}-\frac{1}{4}x^2\right]_0^4+\left[\frac{1}{4}x^2-\frac{2}{3}x^{3/2}\right]_4^9$$

$$=\left[\left(\frac{16}{3}-4\right)-0\right]+\left[\left(\frac{81}{4}-18\right)-\left(4-\frac{16}{3}\right)\right]=\frac{81}{4}+\frac{32}{3}-26=\frac{59}{12}$$





For $4<x<9$

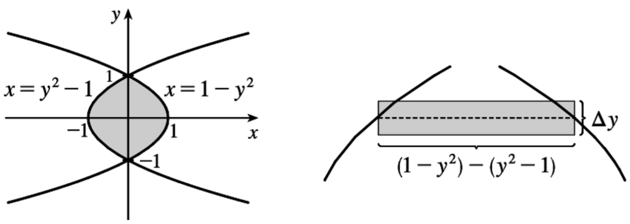
19. The curves intersect when $1-y^2=y^2-1\Leftrightarrow 2=2y^2\Leftrightarrow y^2=1\Leftrightarrow y=\pm 1$.

$$A=\int_{-1}^{1}\left[\left(1-y^2\right)-\left(y^2-1\right)\right]dy$$

$$=\int_{-1}^{1}2\left(1-y^2\right)dy$$

$$=2\cdot 2\int_0^1 \left(1-y^2\right)dy$$

$$=4\left[y-\frac{1}{3}y^3\right]_0^1=4\left(1-\frac{1}{3}\right)=\frac{8}{3}$$

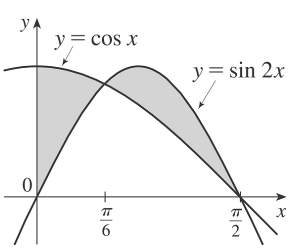


21. Notice that $\cos x=\sin 2x=2\sin x\cos x\Leftrightarrow$
$2\sin x\cos x-\cos x=0\Leftrightarrow\cos x(2\sin x-1)=0\Leftrightarrow$
$2\sin x=1$ or $\cos x=0\Leftrightarrow x=\frac{\pi}{6}$ or $\frac{\pi}{2}$.

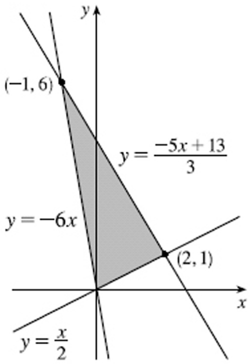
$$A=\int_0^{\pi/6}(\cos x-\sin 2x)\,dx+\int_{\pi/6}^{\pi/2}(\sin 2x-\cos x)\,dx$$

$$=\left[\sin x+\frac{1}{2}\cos 2x\right]_0^{\pi/6}+\left[-\frac{1}{2}\cos 2x-\sin x\right]_{\pi/6}^{\pi/2}$$

$$=\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}-\left(0+\frac{1}{2}\cdot 1\right)+\left(\frac{1}{2}-1\right)-\left(-\frac{1}{2}\cdot\frac{1}{2}-\frac{1}{2}\right)=\frac{1}{2}$$



27.



From the graph, we see that the curves intersect at $x = \pm a \approx \pm 1.02$, with $2\cos x > x^2$ on $(-a, a)$. So the area of the region bounded by the curves is

$$A = \int_{-a}^{a} \left(2\cos x - x^2\right) dx = 2\int_{0}^{a} \left(2\cos x - x^2\right) dx$$

$$= 2\left[2\sin x - \frac{1}{3}x^3\right]_0^a \approx 2.70$$