1. Let $u=3x$. Then $du=3\,dx$, so $dx=\frac{1}{3}\,du$. Thus,
$\int \cos 3x\,dx=\int \cos u\left(\frac{1}{3}\,du\right)=\frac{1}{3}\int \cos u\,du=\frac{1}{3}\sin u+C=\frac{1}{3}\sin 3x+C$. Don't forget that it is often very easy to check an indefinite integration by differentiating your answer. In this case,
$\frac{d}{dx}\left(\frac{1}{3}\sin 3x+C\right)=\frac{1}{3}(\cos 3x)\cdot 3=\cos 3x$, the desired result.

3. Let $u=x^3+1$. Then $du=3x^2\,dx$ and $x^2\,dx=\frac{1}{3}\,du$, so
$\int x^2\sqrt{x^3+1}\,dx=\int \sqrt{u}\left(\frac{1}{3}\,du\right)=\frac{1}{3}\,\frac{u^{3/2}}{3/2}+C=\frac{1}{3}\cdot\frac{2}{3}\,u^{3/2}+C=\frac{2}{9}(x^3+1)^{3/2}+C.$

4. Let $u=\sqrt{x}$. Then $du=\frac{1}{2\sqrt{x}}\,dx$ and $\frac{1}{\sqrt{x}}\,dx=2\,du$, so
$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx=\int \sin u\,(2\,du)=2(-\cos u)+C=-2\cos\sqrt{x}+C.$

7. Let $u=x^2+3$. Then $du=2x\,dx$, so $\int 2x(x^2+3)^4\,dx=\int u^4\,du=\frac{1}{5}u^5+C=\frac{1}{5}(x^2+3)^5+C.$

11. Let $u=1+x+2x^2$. Then $du=(1+4x)\,dx$, so
$\int \frac{1+4x}{\sqrt{1+x+2x^2}}\,dx=\int \frac{du}{\sqrt{u}}=\int u^{-1/2}\,du=\frac{u^{1/2}}{1/2}+C=2\sqrt{1+x+2x^2}+C.$

15. Let $u=4-t$. Then $du=-dt$ and $dt=-du$, so $\int \sqrt{4-t}\,dt=\int u^{1/2}(-du)=-\frac{2}{3}u^{3/2}+C=-\frac{2}{3}(4-t)^{3/2}+C.$

21. Let $u=\sin\theta$. Then $du=\cos\theta\,d\theta$, so $\int \cos\theta\sin^6\theta\,d\theta=\int u^6\,du=\frac{1}{7}u^7+C=\frac{1}{7}\sin^7\theta+C.$

25. Let $u=\cot x$. Then $du=-\csc^2 x\,dx$ and $\csc^2 x\,dx=-du$, so
$\int \sqrt{\cot x}\,\csc^2 x\,dx=\int \sqrt{u}\,(-du)=-\frac{u^{3/2}}{3/2}+C=-\frac{2}{3}(\cot x)^{3/2}+C.$

39. Let $u=1+2x^3$, so $du=6x^2\,dx$. When $x=0$, $u=1$; when $x=1$, $u=3$. Thus,

$$\int_0^1 x^2\left(1+2x^3\right)^5 dx=\int_1^3 u^5\left(\frac{1}{6}\,du\right)=\frac{1}{6}\left[\frac{1}{6}u^6\right]_1^3=\frac{1}{36}\left(3^6-1^6\right)=\frac{1}{36}(729-1)=\frac{728}{36}=\frac{182}{9}$$

40. Let $u=x^2$, so $du=2x\,dx$. When $x=0$, $u=0$; when $x=\sqrt{\pi}$, $u=\pi$. Thus,

$$\int_0^{\sqrt{\pi}} x\cos\left(x^2\right)dx=\int_0^{\pi}\cos u\left(\frac{1}{2}\,du\right)=\frac{1}{2}\left[\sin u\right]_0^{\pi}=\frac{1}{2}(\sin\pi-\sin 0)=\frac{1}{2}(0-0)=0.$$

45. Let $u=\cos\theta$, so $du=-\sin\theta\,d\theta$. When $\theta=0$, $u=1$; when $\theta=\frac{\pi}{3}$, $u=\frac{1}{2}$. Thus,

$$\int_0^{\pi/3}\frac{\sin\theta}{\cos^2\theta}\,d\theta=\int_1^{1/2}\frac{-du}{u^2}=\int_{1/2}^{1}u^{-2}du=\left[-\frac{1}{u}\right]_{1/2}^{1}=-1-(-2)=1.$$

49. Let $u=x-1$, so $u+1=x$ and $du=dx$. When $x=1$, $u=0$; when $x=2$, $u=1$. Thus,

$$\int_1^2 x\sqrt{x-1}\,dx=\int_0^1(u+1)\sqrt{u}\,du=\int_0^1\left(u^{3/2}+u^{1/2}\right)du=\left[\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}\right]_0^1=\frac{2}{5}+\frac{2}{3}=\frac{16}{15}.$$

61. Let $u=2x$. Then $du=2\,dx$, so $\int_0^2 f(2x)\,dx=\int_0^4 f(u)\left(\frac{1}{2}\,du\right)=\frac{1}{2}\int_0^4 f(u)\,du=\frac{1}{2}(10)=5.$

64. The area under the graph of $y=\sin\sqrt{x}$ from 0 to 4 is $A_1=\int_0^4\sin\sqrt{x}\,dx$. The area under the graph of $y=2x\sin x$ from 0 to 2 is $A_2=\int_0^2 2x\sin x\,dx$ $\left[u=x^2,\ du=2x\,dx,\ \sqrt{u}=x \text{ for } 0\le x\le 2\right]$ $=\int_0^4\sin\sqrt{u}\,du$. Since the integration variable is immaterial, $A_1=A_2$.