3. **(a)**

$g(x) = \int_0^x f(t)\,dt$.

$g(0) = \int_0^0 f(t)\,dt = 0$

$g(1) = \int_0^1 f(t)\,dt = 1\cdot 2 = 2$ [rectangle] ,

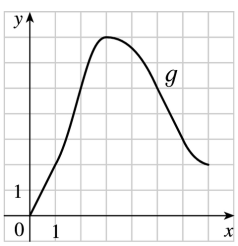
$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt = g(1) + \int_1^2 f(t)\,dt$

$= 2 + 1\cdot 2 + \frac{1}{2}\cdot 1\cdot 2 = 5$ [rectangle plus triangle] ,

$g(3) = \int_0^3 f(t)\,dt = g(2) + \int_2^3 f(t)\,dt = 5 + \frac{1}{2}\cdot 1\cdot 4 = 7$,

$g(6) = g(3) + \int_3^6 f(t)\,dt$ [the integral is negative since f lies under the x–axis]

$= 7 + \left[-\left(\frac{1}{2}\cdot 2\cdot 2 + 1\cdot 2 \right) \right] = 7 - 4 = 3$

(d) 

(b) g is increasing on $(0,3)$ because as x increases from 0 to 3 , we keep adding more area.
(c) g has a maximum value when we start subtracting area; that is, at $x=3$.

15. Let $u=\sqrt{x}$. Then $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$. Also, $\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$, so

$y' = \frac{d}{dx}\int_3^{\sqrt{x}} \frac{\cos t}{t}\,dt = \frac{d}{du}\int_3^{u} \frac{\cos t}{t}\,dt\cdot \frac{du}{dx} = \frac{\cos u}{u}\cdot \frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{\sqrt{x}}\cdot \frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{2x}$.

21. $\int_2^8 (4x+3)\,dx = \left[\frac{4}{2}x^2 + 3x \right]_2^8 = \left(2\cdot 8^2 + 3\cdot 8\right) - \left(2\cdot 2^2 + 3\cdot 2\right) = 152 - 14 = 138$

23. $\int_0^1 x^{4/5}\,dx = \left[\frac{5}{9}x^{9/5} \right]_0^1 = \frac{5}{9} - 0 = \frac{5}{9}$

27. $\int_{-5}^{5} \frac{2}{x^3}\, dx$ does not exist because the function $f(x)=\frac{2}{x^3}$ has an infinite discontinuity at $x=0$; that is, f is discontinuous on the interval $[-5,5]$.

29. $\int_0^2 x(2+x^5)\,dx=\int_0^2 (2x+x^6)\,dx=\left[x^2+\frac{1}{7}x^7\right]_0^2=\left(4+\frac{128}{7}\right)-(0+0)=\frac{156}{7}$

31. $\int_0^{\pi/4} \sec^2 t\,dt=[\tan t]_0^{\pi/4}=\tan\frac{\pi}{4}-\tan 0=1-0=1$

48. For the curve to be concave upward, we must have $y''>0$. $y=\int_0^x \frac{1}{1+t+t^2}\,dt \Rightarrow y'=\frac{1}{1+x+x^2} \Rightarrow$ $y''=\frac{-(1+2x)}{\left(1+x+x^2\right)^2}$. For this expression to be positive, we must have $(1+2x)<0$, since $\left(1+x+x^2\right)^2>0$ for all x. $(1+2x)<0 \Leftrightarrow x<-\frac{1}{2}$. Thus, the curve is concave upward on $\left(-\infty,-\frac{1}{2}\right)$.

53. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}=\lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3=\int_0^1 x^3\,dx=\left[\frac{x^4}{4}\right]_0^1=\frac{1}{4}$

58. **(a)** If $x<0$, then $g(x)=\int_0^x f(t)\,dt=\int_0^x 0\,dt=0$.

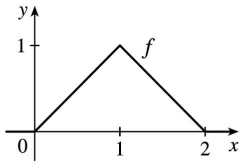
If $0\le x\le 1$, then $g(x)=\int_0^x f(t)\,dt=\int_0^x t\,dt=\left[\frac{1}{2}t^2\right]_0^x=\frac{1}{2}x^2$.

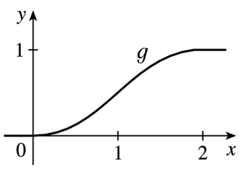
If $1<x\le 2$, then

$$\begin{aligned} g(x) &=\int_0^x f(t)\,dt=\int_0^1 f(t)\,dt+\int_1^x f(t)\,dt \\ &=g(1)+\int_1^x (2-t)\,dt=\frac{1}{2}(1)^2+\left[2t-\frac{1}{2}t^2\right]_1^x \\ &=\frac{1}{2}+\left(2x-\frac{1}{2}x^2\right)-\left(2-\frac{1}{2}\right)=2x-\frac{1}{2}x^2-1 . \end{aligned}$$

If $x>2$, then $g(x)=\int_0^x f(t)\,dt=g(2)+\int_2^x 0\,dt=1+0=1$. So

$$g(x)=\begin{cases} 0 & \text{if } x<0 \\ \frac{1}{2}x^2 & \text{if } 0\leq x\leq 1 \\ 2x-\frac{1}{2}x^2-1 & \text{if } 1<x\leq 2 \\ 1 & \text{if } x>2 \end{cases}$$

(b) 



(c) f is not differentiable at its corners at $x=0$, 1 , and 2. f is differentiable on $(-\infty,0)$, $(0,1)$, $(1,2)$ and $(2,\infty)$. g is differentiable on $(-\infty,\infty)$.