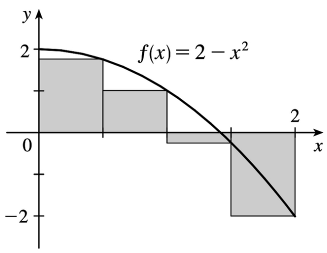
1.

$$R_4 = \sum_{i=1}^{4} f\left(x_i\right) \Delta x \quad \left[x_i^* = x_i \text{ is a right endpoint and } \Delta x = 0.5\right]$$
$$= 0.5[f(0.5) + f(1) + f(1.5) + f(2)] \quad \left[f(x) = 2 - x^2\right]$$
$$= 0.5[1.75 + 1 + (-0.25) + (-2)]$$
$$= 0.5(0.5) = 0.25$$

The Riemann sum represents the sum of the areas of the two rectangles above the x –axis minus the sum of the areas of the two rectangles below the x –axis; that is, the *net area* of the rectangles with respect to the x –axis.



5. $\Delta x = (b - a)/n = (8 - 0)/4 = 8/4 = 2$.

(a) Using the right endpoints to approximate $\int_0^8 f(x)\,dx$, we have

$\sum_{i=1}^{4} f\left(x_i\right) \Delta x = 2[f(2) + f(4) + f(6) + f(8)] \approx 2[1 + 2 + (-2) + 1] = 4$.

(b) Using the left endpoints to approximate $\int_0^8 f(x)\,dx$, we have

$\sum_{i=1}^{4} f\left(x_{i-1}\right) \Delta x = 2[f(0) + f(2) + f(4) + f(6)] \approx 2[2 + 1 + 2 + (-2)] = 6$.

(c) Using the midpoint of each subinterval to approximate $\int_0^8 f(x)\,dx$, we have

$\sum_{i=1}^{4} f\left(\overline{x}_i\right) \Delta x = 2[f(1) + f(3) + f(5) + f(7)] \approx 2[3 + 2 + 1 + (-1)] = 10$.

7. Since f is increasing, $L_5 \le \int_0^{25} f(x)\,dx \le R_5$.

$$\text{Lower estimate} = L_5 = \sum_{i=1}^{5} f\left(x_{i-1}\right) \Delta x = 5[f(0) + f(5) + f(10) + f(15) + f(20)]$$
$$= 5(-42 - 37 - 25 - 6 + 15) = 5(-95) = -475$$

$$\text{Upper estimate} = R_5 = \sum_{i=1}^{5} f\left(x_i\right) \Delta x = 5[f(5) + f(10) + f(15) + f(20) + f(25)]$$
$$= 5(-37 - 25 - 6 + 15 + 36) = 5(-17) = -85$$

17. On $[0,\pi]$, $\lim_{n\to\infty}\sum_{i=1}^{n} x_i \sin x_i \, \Delta x = \int_0^{\pi} x \sin x \, dx$.

23. Note that $\Delta x = \frac{2-0}{n} = \frac{2}{n}$ and $x_i = 0 + i\,\Delta x = \frac{2i}{n}$.

$$\int_0^2 \left(2-x^2\right) dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(x_i\right)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left(2-\frac{4i^2}{n^2}\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left[\sum_{i=1}^{n} 2 - \frac{4}{n^2}\sum_{i=1}^{n} i^2\right] = \lim_{n\to\infty}\frac{2}{n}\left(2n - \frac{4}{n^2}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty}\left[4 - \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right] = \lim_{n\to\infty}\left(4 - \frac{4}{3}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}\right)$$

$$= \lim_{n\to\infty}\left[4 - \frac{4}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 4 - \frac{4}{3}\cdot 1\cdot 2 = \frac{4}{3}$$

29. $f(x) = \frac{x}{1+x^5}$, $a=2$, $b=6$, and $\Delta x = \frac{6-2}{n} = \frac{4}{n}$. Using Equation 3, we get $x_i^* = x_i = 2 + i\,\Delta x = 2 + \frac{4i}{n}$, so

$$\int_2^6 \frac{x}{1+x^5}\, dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2+\frac{4i}{n}}{1+\left(2+\frac{4i}{n}\right)^5}\cdot\frac{4}{n}.$$

33. **(a)** Think of $\int_0^2 f(x)\,dx$ as the area of a trapezoid with bases 1 and 3 and height 2. The area of a trapezoid is $A = \frac{1}{2}(b+B)h$, so $\int_0^2 f(x)\,dx = \frac{1}{2}(1+3)2 = 4$.

(b)

$$\int_0^5 f(x)\,dx = \underbrace{\int_0^2 f(x)\,dx}_{\text{trapezoid}} + \underbrace{\int_2^3 f(x)\,dx}_{\text{rectangle}} + \underbrace{\int_3^5 f(x)\,dx}_{\text{triangle}}$$

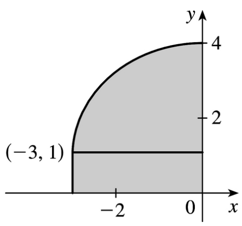
$$= \frac{1}{2}(1+3)2 + 3\cdot 1 + \frac{1}{2}\cdot 2\cdot 3 = 4+3+3 = 10$$

(c) $\int_5^7 f(x)\,dx$ is the negative of the area of the triangle with base 2 and height 3.

$\int_5^7 f(x)\,dx = -\frac{1}{2}\cdot 2\cdot 3 = -3$.

(d) $\int_7^9 f(x)\,dx$ is the negative of the area of a trapezoid with bases 3 and 2 and height 2 , so it equals $-\frac{1}{2}(B+b)h=-\frac{1}{2}(3+2)2=-5$. Thus, $\int_0^9 f(x)\,dx=\int_0^5 f(x)\,dx+\int_5^7 f(x)\,dx+\int_7^9 f(x)\,dx=10+(-3)+(-5)=2$.

37. $\int_{-3}^0 \left(1+\sqrt{9-x^2}\right)dx$ can be interpreted as the area under the graph of $f(x)=1+\sqrt{9-x^2}$ between $x=-3$ and $x=0$. This is equal to one–quarter the area of the circle with radius 3 , plus the area of the rectangle, so $\int_{-3}^0 \left(1+\sqrt{9-x^2}\right)dx=\frac{1}{4}\pi\cdot 3^2+1\cdot 3=3+\frac{9}{4}\pi$.



49. $\int_0^9 [2f(x)+3g(x)]\,dx=2\int_0^9 f(x)\,dx+3\int_0^9 g(x)\,dx=2(37)+3(16)=122$

50. If $f(x)=\begin{cases} 3 & \text{for } x<3 \\ x & \text{for } x\geq 3 \end{cases}$, then $\int_0^5 f(x)\,dx$ can be interpreted as the area of the shaded region, which consists of a 5–by–3 rectangle surmounted by an isosceles right triangle whose legs have length 2. Thus, $\int_0^5 f(x)\,dx=5(3)+\frac{1}{2}(2)(2)=17$.