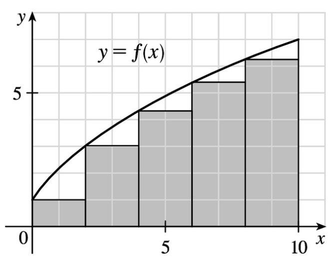
1. **(a)** Since f is *increasing* , we can obtain a *lower* estimate by using *left* endpoints. We are instructed to use five rectangles, so *n*=5.

$$L_5 = \sum_{i=1}^{5} f\left(x_{i-1}\right)\Delta x \qquad \left[\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2\right]$$

$$= f\left(x_0\right)\cdot 2 + f\left(x_1\right)\cdot 2 + f\left(x_2\right)\cdot 2 + f\left(x_3\right)\cdot 2 + f\left(x_4\right)\cdot 2$$

$$= 2\left[f(0)+f(2)+f(4)+f(6)+f(8)\right]$$

t;\approx2(1+3+4.3+5.4+6.3)=2(20)=40



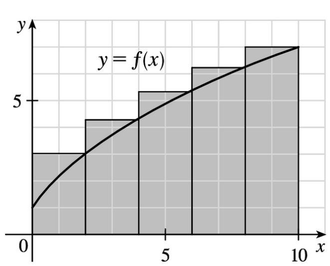
Since f is *increasing* , we can obtain an *upper* estimate by using *right* endpoints.

$$R_5 = \sum_{i=1}^{5} f\left(x_i\right)\Delta x$$

$$= 2\left[f\left(x_1\right)+f\left(x_2\right)+f\left(x_3\right)+f\left(x_4\right)+f\left(x_5\right)\right]$$

$$= 2\left[f(2)+f(4)+f(6)+f(8)+f(10)\right]$$

$$\approx 2(3+4.3+5.4+6.3+7)=2(26)=52$$

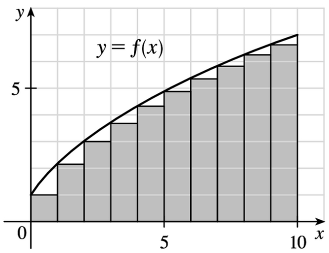


Comparing R_5 to L_5 , we see that we have added the area of the rightmost upper rectangle, $f(10)\cdot 2$, to the sum and subtracted the area of the leftmost lower rectangle, $f(0)\cdot 2$, from the sum.

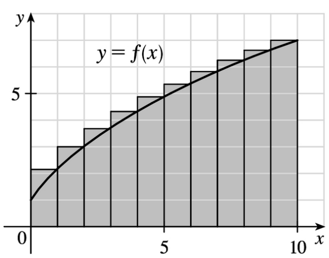
(b)

$$L_{10} = \sum_{i=1}^{10} f\left(x_{i-1}\right)\Delta x \qquad \left[\Delta x = \frac{10-0}{10} = 1\right]$$

$$=1\left[f\left(x_0\right)+f\left(x_1\right)+\cdots+f\left(x_9\right)\right]$$
$$=f(0)+f(1)+\cdots+f(9)$$
$$\approx 1+2.1+3+3.7+4.3+4.9+5.4+5.8+6.3+6.7$$
$$=43.2$$



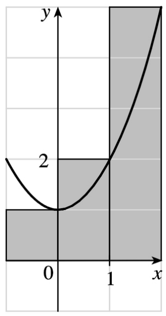
$$R_{10}=\sum_{i=1}^{10} f\left(x_i\right)\Delta x=f(1)+f(2)+\cdots+f(10)$$
$$=L_{10}+1\cdot f(10)-1\cdot f(0)$$ [add rightmost upper rectangle, subtract leftmost lower rectangle]
$$=43.2+7-1=49.2$$

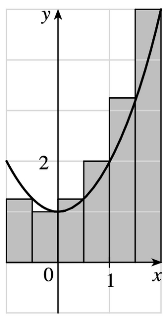


5. **(a)** $f(x)=1+x^2$ and $\Delta x=\dfrac{2-(-1)}{3}=1\Rightarrow R_3=1\cdot f(0)+1\cdot f(1)+1\cdot f(2)=1\cdot 1+1\cdot 2+1\cdot 5=8$.

$\Delta x=\dfrac{2-(-1)}{6}=0.5\Rightarrow$

$$R_6=0.5[f(-0.5)+f(0)+f(0.5)+f(1)+f(1.5)+f(2)]$$
$$=0.5(1.25+1+1.25+2+3.25+5)$$
$$=0.5(13.75)=6.875$$





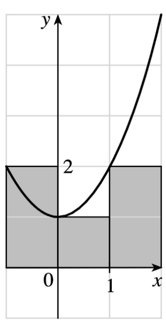
(b)

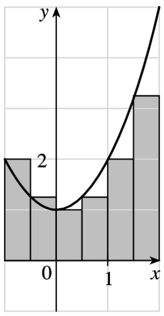
$L_3 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) = 1 \cdot 2 + 1 \cdot 1 + 1 \cdot 2 = 5$

$L_6 = 0.5[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)]$

$= 0.5(2 + 1.25 + 1 + 1.25 + 2 + 3.25)$

$= 0.5(10.75) = 5.375$

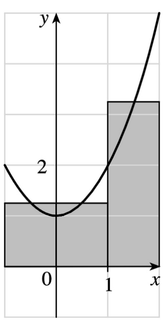


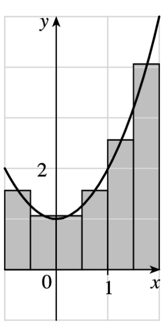


(c)

$$M_3 = 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5)$$
$$= 1 \cdot 1.25 + 1 \cdot 1.25 + 1 \cdot 3.25 = 5.75$$

$$M_6 = 0.5[f(-0.75) + f(-0.25) + f(0.25)$$
$$+ f(0.75) + f(1.25) + f(1.75)]$$
$$= 0.5(1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625)$$
$$= 0.5(11.875) = 5.9375$$





(d) M_6 appears to be the best estimate.

11. Since v is an increasing function, L_6 will give us a lower estimate and R_6 will give us an upper estimate.

$$L_6 = (0\text{ft/s})(0.5\text{s}) + (6.2)(0.5) + (10.8)(0.5) + (14.9)(0.5) + (18.1)(0.5) + (19.4)(0.5)$$
$$= 0.5(69.4) = 34.7 \text{ ft}$$

$$R_6 = 0.5(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2) = 0.5(89.6) = 44.8 \text{ ft}$$

15. For a decreasing function, using left endpoints gives us an overestimate and using right endpoints results in an underestimate. We will use M_6 to get an estimate. $\Delta t = 1$, so

$$M_6 = 1[v(0.5) + v(1.5) + v(2.5) + v(3.5) + v(4.5) + v(5.5)]$$
$$\approx 55 + 40 + 28 + 18 + 10 + 4 = 155\text{ft}$$

For a very rough check on the above calculation, we can draw a line from $(0,70)$ to $(6,0)$ and calculate the area of the triangle: $\frac{1}{2}(70)(6) = 210$. This is clearly an overestimate, so our midpoint estimate of 155 is reasonable.

19. $f(x) = x\cos x$, $0 \le x \le \frac{\pi}{2}$. $\Delta x = (\frac{\pi}{2} - 0)/n = \frac{\pi}{2}/n$ and $x_i = 0 + i\,\Delta x = \frac{\pi}{2}\,i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i\right)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i\pi}{2n} \cos\left(\frac{i\pi}{2n}\right) \cdot \frac{\pi}{2n} .$$

20. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10}$ can be interpreted as the area of the region lying under the graph of $y = (5+x)^{10}$ on the interval $[0,2]$, since for $y = (5+x)^{10}$ on $[0,2]$ with $\Delta x = \frac{2-0}{n} = \frac{2}{n}$, $x_i = 0 + i\,\Delta x = \frac{2i}{n}$, and $x_i^* = x_i$, the expression for the area is $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(5 + \frac{2i}{n}\right)^{10} \frac{2}{n}$. Note that the answer is not unique. We could use $y = x^{10}$ on $[5,7]$ or, in general, $y = ((5-n)+x)^{10}$ on $[n, n+2]$.