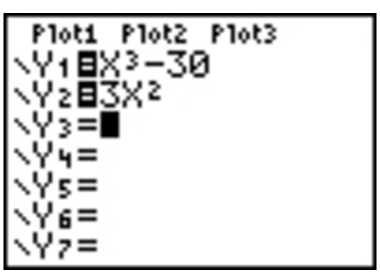
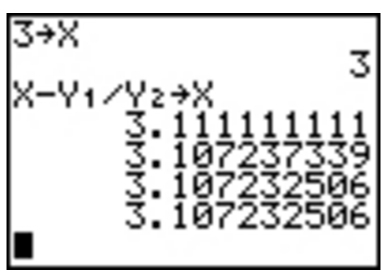
3. Since $x_1=3$ and $y=5x-4$ is tangent to $y=f(x)$ at $x=3$, we simply need to find where the tangent line intersects the $x-$ axis. $y=0 \Rightarrow 5x_2-4=0 \Rightarrow x_2=\frac{4}{5}$.

5. $f(x)=x^3+2x-4 \Rightarrow f^{/}(x)=3x^2+2$, so $x_{n+1}=x_n-\frac{x_n^3+2x_n-4}{3x_n^2+2}$. Now $x_1=1 \Rightarrow x_2=1-\frac{1+2-4}{3\cdot 1^2+2}=1-\frac{-1}{5}=1.2 \Rightarrow$ $x_3=1.2-\frac{(1.2)^3+2(1.2)-4}{3(1.2)^2+2} \approx 1.1797$.

11. To approximate $x=\sqrt[3]{30}$ (so that $x^3=30$), we can take $f(x)=x^3-30$. So $f^{/}(x)=3x^2$, and thus, $x_{n+1}=x_n-\frac{x_n^3-30}{3x_n^2}$. Since $\sqrt[3]{27}=3$ and 27 is close to 30 , we'll use $x_1=3$. We need to find approximations until they agree to eight decimal places. $x_1=3 \Rightarrow x_2 \approx 3.11111111$, $x_3 \approx 3.10723734$, $x_4 \approx 3.10723251 \approx x_5$. So $\sqrt[3]{30} \approx 3.10723251$, to eight decimal places. Here is a quick and easy method for finding the iterations for Newton's method on a programmable calculator. (The screens shown are from the TI–83 Plus, but the method is similar on other calculators.) Assign $f(x)=x^3-30$ to Y_1 , and $f^{/}(x)=3x^2$ to Y_2. Now store $x_1=3$ in X and then enter $X-Y_1/Y_2 \rightarrow X$ to get $x_2=3.\overline{1}$. By successively pressing the ENTER key, you get the approximations x_3 , x_4 , $\ldots$.



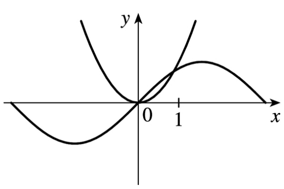


In Derive, load the utility file NEWTON$(x\hat{}3-30,x,3)$ and then APPROXIMATE to get . You can request a specific iteration by adding a fourth argument. For example, NEWTON$(x\hat{}3-30,x,3,\ 2)$ gives [3,3.11111111,3.10723733] .
In Maple, make the assignments f:=$x \rightarrow x\hat{}3-30$; , g:=$x \rightarrow x-f(x)/D(f)(x)$; , and x:=3.;. Repeatedly execute the command x:=$g(x)$; to generate successive approximations.

In Mathematica, make the assignments $f[x]:=x\wedge 3-30$, $g[x]:=x-f[x]/f^{/}[x]$, and $x=3$. Repeatedly execute the command $x=g[x]$ to generate successive approximations.

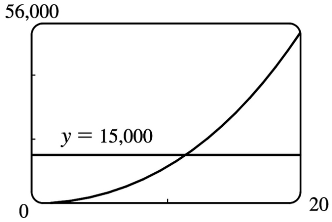
13. $f(x)=2x^3-6x^2+3x+1 \Rightarrow f^{/}(x)=6x^2-12x+3 \Rightarrow x_{n+1}=x_n-\dfrac{2x_n^3-6x_n^2+3x_n+1}{6x_n^2-12x_n+3}$. We need to find approximations until they agree to six decimal places. $x_1=2.5 \Rightarrow x_2 \approx 2.285714$, $x_3 \approx 2.228824$, $x_4 \approx 2.224765$, $x_5 \approx 2.224745 \approx x_6$. So the root is 2.224745 , to six decimal places.

15. $\sin x=x^2$, so $f(x)=\sin x-x^2 \Rightarrow f^{/}(x)=\cos x-2x \Rightarrow x_{n+1}=x_n-\dfrac{\sin x_n-x_n^2}{\cos x_n-2x_n}$. From the figure, the positive root of $\sin x=x^2$ is near 1.$x_1=1 \Rightarrow x_2 \approx 0.891396$, $x_3 \approx 0.876985$, $x_4 \approx 0.876726 \approx x_5$. So the positive root is 0.876726 , to six decimal places.



29. $f(x)=x^3-3x+6 \Rightarrow f^{/}(x)=3x^2-3$. If $x_1=1$, then $f^{/}\left(x_1\right)=0$ and the tangent line used for approximating x_2 is horizontal. Attempting to find x_2 results in trying to divide by zero.

37.



The volume of the silo, in terms of its radius, is $V(r)=\pi r^2(30)+\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)=30\pi r^2+\frac{2}{3}\pi r^3$. From a graph of V , we see that $V(r)=15,000$ at $r \approx 11$ ft. Now we use Newton's method to solve the equation $V(r)-15,000=0$. $\dfrac{dV}{dr}=60\pi r+2\pi r^2$, so $r_{n+1}=r_n-\dfrac{30\pi r_n^2+\frac{2}{3}\pi r_n^3-15{,}000}{60\pi r_n+2\pi r_n^2}$. Taking $r_1=11$, we get $r_2 \approx 11.2853$, $r_3 \approx 11.2807 \approx r_4$. So in order for the silo to hold 15 , 000 ft^3 of grain, its radius must be about 11.2807 ft.