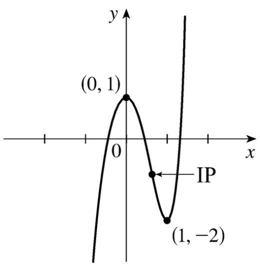
7. $y=f(x)=2x^5-5x^2+1$ **A.** $D=R$ **B.** $y-$ intercept: $f(0)=1$ **C.** No symmetry **D.** No asymptote **E.** $f'(x)=10x^4-10x=10x(x^3-1)=10x(x-1)(x^2+x+1)$, so $f'(x)<0 \Leftrightarrow 0<x<1$ and $f'(x)>0 \Leftrightarrow x<0$ or $x>1$. Thus, f is increasing on $(-\infty,0)$ and $(1,\infty)$ and decreasing on $(0,1)$.

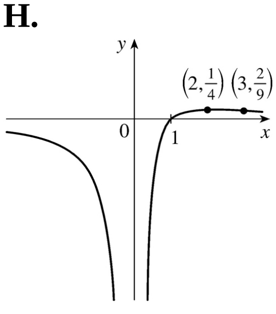
F. Local maximum value $f(0)=1$, local minimum value $f(1)=-2$ **G.** $f''(x)=40x^3-10=10(4x^3-1)$ so $f''(x)=0 \Leftrightarrow x=1/\sqrt[3]{4}$. $f''(x)>0 \Leftrightarrow x>1/\sqrt[3]{4}$ and $f''(x)<0 \Leftrightarrow x<1/\sqrt[3]{4}$, < so f is CD on $\left(-\infty,1/\sqrt[3]{4}\right)$ and CU on $\left(1/\sqrt[3]{4},\infty\right)$. IP at $\left(\dfrac{1}{\sqrt[3]{4}},1-\dfrac{9}{2\left(\sqrt[3]{4}\right)^2}\right)\approx(0.630,-0.786)$

H.



15. $y=f(x)=\dfrac{x-1}{x^2}$ **A.** $D=\{x\,|\,x\neq 0\}=(-\infty,0)\cup(0,\infty)$ **B.** No y –intercept; x –intercept: $f(x)=0 \Leftrightarrow x=1$ **C.** No symmetry **D.** $\lim\limits_{x\to\pm\infty}\dfrac{x-1}{x^2}=0$, so $y=0$ is a HA. $\lim\limits_{x\to 0}\dfrac{x-1}{x^2}=-\infty$, so $x=0$ is a VA. **E.** $f'(x)=\dfrac{x^2\cdot 1-(x-1)\cdot 2x}{\left(x^2\right)^2}=\dfrac{-x^2+2x}{x^4}=\dfrac{-(x-2)}{x^3}$, so $f'(x)>0 \Leftrightarrow 0<x<2$ and $f'(x)<0 \Leftrightarrow x<0$ or $x>2$. Thus, f is increasing on $(0,2)$ and decreasing on $(-\infty,0)$ and $(2,\infty)$. **F.** No local minimum, local maximum value $f(2)=\dfrac{1}{4}$. **G.** $f''(x)=\dfrac{x^3\cdot(-1)-[-(x-2)]\cdot 3x^2}{\left(x^3\right)^2}=\dfrac{2x^3-6x^2}{x^6}=\dfrac{2(x-3)}{x^4}$. $f''(x)$ is negative on $(-\infty,0)$ and $(0,3)$ and positive on $(3,\infty)$, so f is CD on $(-\infty,0)$ and $(0,3)$ and CU on $(3,\infty)$. IP at $\left(3,\dfrac{2}{9}\right)$

H.



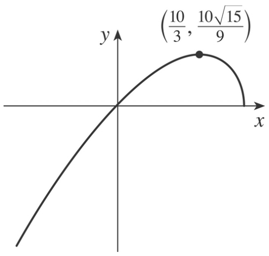
19. $y=f(x)=x\sqrt{5-x}$ **A.** The domain is $\{x|5-x\geq 0\}=(-\infty,5]$ **B.** $y-$ intercept: $f(0)=0$; x –intercepts: $f(x)=0\Leftrightarrow x=0$, 5 **C.** No symmetry **D.** No asymptote

E. $f^{/}(x)=x\cdot\frac{1}{2}(5-x)^{-1/2}(-1)+(5-x)^{1/2}\cdot 1=\frac{1}{2}(5-x)^{-1/2}[-x+2(5-x)]=\frac{10-3x}{2\sqrt{5-x}}>0\Leftrightarrow x<\frac{10}{3}$, so f is increasing on $\left(-\infty,\frac{10}{3}\right)$ and decreasing on $\left(\frac{10}{3},5\right)$. **F.** Local maximum value $f\left(\frac{10}{3}\right)=\frac{10}{9}\sqrt{15}\approx 4.3$; no local minimum

G. $f^{//}(x)=\frac{2(5-x)^{1/2}(-3)-(10-3x)\cdot 2\left(\frac{1}{2}\right)(5-x)^{-1/2}(-1)}{\left(2\sqrt{5-x}\right)^{2}}=\frac{(5-x)^{-1/2}[-6(5-x)+(10-3x)]}{4(5-x)}=\frac{3x-20}{4(5-x)^{3/2}}$

$f^{//}(x)<0$ for $x<5$, so f is CD on $(-\infty,5)$. No IP

H.



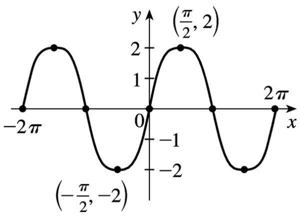
31. $y=f(x)=3\sin x-\sin^{3}x$ **A.** $D=R$ **B.** $y-$ intercept: $f(0)=0$; x –intercepts: $f(x)=0\Rightarrow\sin x(3-\sin^{2}x)=0\Rightarrow\sin x=0$ [since $\sin^{2}x\leq 1<3$] $\Rightarrow x=n\pi$, n an integer.

C. $f(-x)=-f(x)$, so f is odd; the graph (shown for $-2\pi\leq x\leq 2\pi$) is symmetric about the origin and periodic with period 2π. **D.** No asymptote **E.** $f^{/}(x)=3\cos x-3\sin^{2}x\cos x=3\cos x\left(1-\sin^{2}x\right)=3\cos^{3}x$. $f^{/}(x)>0\Leftrightarrow\cos x>0\Leftrightarrow x\in\left(2n\pi-\frac{\pi}{2},2n\pi+\frac{\pi}{2}\right)$ [hence CD on the intervals $(2n\pi,2(n+1)\pi)$] for each integer n , and $f^{/}(x)<0\Leftrightarrow$ $\cos x<0\Leftrightarrow x\in\left(2n\pi+\frac{\pi}{2},2n\pi+\frac{3\pi}{2}\right)$ [hence CD on the intervals $((2n-1)\pi,2n\pi)$] for each integer n. Thus, f is increasing on $\left(2n\pi-\frac{\pi}{2},2n\pi+\frac{\pi}{2}\right)$ for each integer n , and f is decreasing on $\left(2n\pi+\frac{\pi}{2},2n\pi+\frac{3\pi}{2}\right)$ for each integer n. **F.** f has local maximum values $f(2n\pi+\frac{\pi}{2})=2$ and local minimum values $f(2n\pi+\frac{3\pi}{2})=-2$.

G. $f^{//}(x)=-9\sin x\cos^{2}x=-9\sin x(1-\sin^{2}x)=-9\sin x(1-\sin x)(1+\sin x)$. $f^{//}(x)<0\Leftrightarrow\sin x>0$ and $\sin x\neq\pm 1\Leftrightarrow x\in\left(2n\pi,2n\pi+\frac{\pi}{2}\right)\cup\left(2n\pi+\frac{\pi}{2},2n\pi+\pi\right)$ for some integer n. $f^{//}(x)>0\Leftrightarrow\sin x<0$ and

$\sin x \neq \pm 1 \Leftrightarrow x \in \left((2n-1)\pi, (2n-1)\pi + \frac{\pi}{2} \right) \cup \left((2n-1)\pi + \frac{\pi}{2}, 2n\pi \right)$ for some integer n. Thus, f is CD on the intervals $\left(2n\pi, \left(2n + \frac{1}{2} \right)\pi \right)$ and $\left(\left(2n + \frac{1}{2} \right)\pi, (2n+1)\pi \right)$ for each integer n, and f is CU on the intervals $\left((2n-1)\pi, \left(2n - \frac{1}{2} \right)\pi \right)$ and $\left(\left(2n - \frac{1}{2} \right)\pi, 2n\pi \right)$ for each integer n. f has inflection points at $(n\pi, 0)$ for each integer n.

H.



51. $y = f(x) = \frac{2x^3 + x^2 + 1}{x^2 + 1} = 2x + 1 + \frac{-2x}{x^2 + 1}$ **A.** $D = R$ **B.** y-intercept: $f(0) = 1$; x-intercept: $f(x) = 0 \Rightarrow 0 = 2x^3 + x^2 + 1 = (x+1)(2x^2 - x + 1) \Rightarrow x = -1$ **C.** No symmetry **D.** No VA $\lim_{x \to \pm\infty} [f(x) - (2x+1)] = \lim_{x \to \pm\infty} \frac{-2x}{x^2 + 1} = \lim_{x \to \pm\infty} \frac{-2/x}{1 + 1/x^2} = 0$, so the line $y = 2x + 1$ is a slant asymptote. **E.**

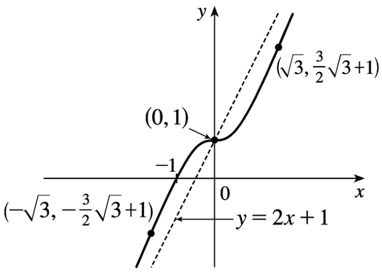
$$f'(x) = 2 + \frac{\left(x^2+1\right)(-2) - (-2x)(2x)}{\left(x^2+1\right)^2} = \frac{2\left(x^4 + 2x^2 + 1\right) - 2x^2 - 2 + 4x^2}{\left(x^2+1\right)^2} = \frac{2x^4 + 6x^2}{(x^2+1)^2} = \frac{2x^2(x^2+3)}{(x^2+1)^2}$$

so $f'(x) > 0$ if $x \neq 0$. Thus, f is increasing on $(-\infty, 0)$ and $(0, \infty)$. Since f is continuous at 0, f is increasing on R. **F.** No extreme values.

G. $$f''(x) = \frac{\left(x^2+1\right)^2 \cdot \left(8x^3 + 12x\right) - \left(2x^4 + 6x^2\right) \cdot 2\left(x^2+1\right)(2x)}{\left[\left(x^2+1\right)^2\right]^2} = \frac{4x\left(x^2+1\right)\left[\left(x^2+1\right)\left(2x^2+3\right) - 2x^4 - 6x^2\right]}{\left(x^2+1\right)^4}$$

$= \frac{4x\left(-x^2+3\right)}{\left(x^2+1\right)^3}$ so $f''(x) > 0$ for $x < -\sqrt{3}$ and $0 < x < \sqrt{3}$, and $f''(x) < 0$ for $-\sqrt{3} < x < 0$ and $x > \sqrt{3}$. f is CU on $\left(-\infty, -\sqrt{3}\right)$ and $\left(0, \sqrt{3}\right)$, and CD on $\left(-\sqrt{3}, 0\right)$ and $\left(\sqrt{3}, \infty\right)$. There are three IPs: $(0, 1)$, $\left(-\sqrt{3}, -\frac{3}{2}\sqrt{3} + 1 \right) \approx (-1.73, -1.60)$, and $\left(\sqrt{3}, \frac{3}{2}\sqrt{3} + 1 \right) \approx (1.73, 3.60)$.

H.


y
(√3, 3/2 √3+1)
(0, 1)
−1
0
x
(−√3, −3/2 √3+1)
y = 2x + 1