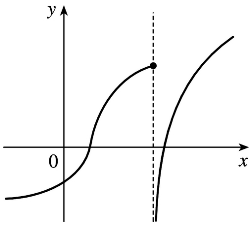
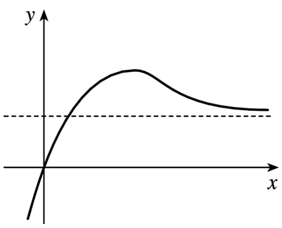
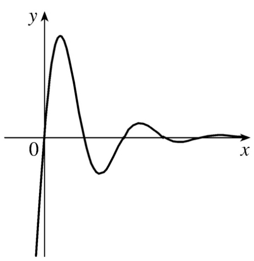
2. **(a)** The graph of a function can intersect a vertical asymptote in the sense that it can meet but not cross it.

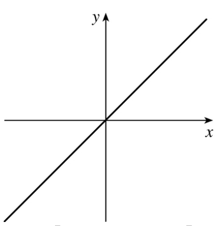


The graph of a function can intersect a horizontal asymptote. It can even intersect its horizontal asymptote an infinite number of times.

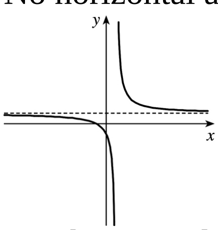




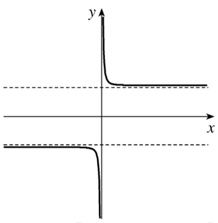
(b) The graph of a function can have 0 , 1 , or 2 horizontal asymptotes. Representative examples are shown.



No horizontal asymptote



One horizontal asymptote



Two horizontal asymptotes

4.

(a) $\lim_{x \to \infty} g(x) = 2$

(b) $\lim_{x \to -\infty} g(x) = -2$

(c) $\lim_{x \to 3} g(x) = \infty$

(d) $\lim_{x \to 0} g(x) = -\infty$

(e) $\lim_{x \to -2^+} g(x) = -\infty$

(f) Vertical: $x=-2$, $x=0$, $x=3$; Horizontal: $y=-2$, $y=2$

5. If $f(x)=x^2/2^x$, then a calculator gives $f(0)=0$, $f(1)=0.5$, $f(2)=1$, $f(3)=1.125$, $f(4)=1$, $f(5)=0.78125$, $f(6)=0.5625$, $f(7)=0.3828125$, $f(8)=0.25$, $f(9)=0.158203125$, $f(10)=0.09765625$, $f(20)\approx 0.00038147$, $f(50)\approx 2.2204\times 10^{-12}$, $f(100)\approx 7.8886\times 10^{-27}$.
It appears that $\lim_{x \to \infty} \left(x^2/2^x\right)=0.$

13. Divide both the numerator and denominator by x^3 (the highest power of x that occurs in the denominator).

$$\lim_{x \to \infty} \frac{x^3+5x}{2x^3-x^2+4} = \lim_{x \to \infty} \frac{\dfrac{x^3+5x}{x^3}}{\dfrac{2x^3-x^2+4}{x^3}} = \lim_{x \to \infty} \frac{1+\dfrac{5}{x^2}}{2-\dfrac{1}{x}+\dfrac{4}{x^3}} = \frac{\lim_{x \to \infty}\left(1+\dfrac{5}{x^2}\right)}{\lim_{x \to \infty}\left(2-\dfrac{1}{x}+\dfrac{4}{x^3}\right)}$$

$$= \frac{\lim_{x\to\infty} 1 + 5\lim_{x\to\infty} \frac{1}{x^2}}{\lim_{x\to\infty} 2 - \lim_{x\to\infty} \frac{1}{x} + 4\lim_{x\to\infty} \frac{1}{x^3}} = \frac{1+5(0)}{2-0+4(0)} = \frac{1}{2}$$

15. First, multiply the factors in the denominator. Then divide both the numerator and denominator by u^4.

$$\lim_{u\to\infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)} = \lim_{u\to\infty} \frac{4u^4+5}{2u^4-5u^2+2} = \lim_{u\to\infty} \frac{\frac{4u^4+5}{u^4}}{\frac{2u^4-5u^2+2}{u^4}} = \lim_{u\to\infty} \frac{4+\frac{5}{u^4}}{2-\frac{5}{u^2}+\frac{2}{u^4}}$$

$$= \frac{\lim_{u\to\infty}\left(4+\frac{5}{u^4}\right)}{\lim_{u\to\infty}\left(2-\frac{5}{u^2}+\frac{2}{u^4}\right)} = \frac{\lim_{u\to\infty} 4 + 5\lim_{u\to\infty} \frac{1}{u^4}}{\lim_{u\to\infty} 2 - 5\lim_{u\to\infty} \frac{1}{u^2} + 2\lim_{u\to\infty} \frac{1}{u^4}} = \frac{4+5(0)}{2-5(0)+2(0)}$$

$$= \frac{4}{2} = 2$$

17.

$$\lim_{x\to\infty} \frac{\sqrt{9x^6-x}}{x^3+1} = \lim_{x\to\infty} \frac{\sqrt{9x^6-x}\,/x^3}{(x^3+1)/x^3} = \frac{\lim_{x\to\infty} \sqrt{(9x^6-x)/x^6}}{\lim_{x\to\infty} (1+1/x^3)} \qquad [\text{ since } x^3 = \sqrt{x^6} \text{ for } x>0 \text{ }]$$

$$= \frac{\lim_{x\to\infty} \sqrt{9-1/x^5}}{\lim_{x\to\infty} 1 + \lim_{x\to\infty} (1/x^3)} = \frac{\sqrt{\lim_{x\to\infty} 9 - \lim_{x\to\infty} (1/x^5)}}{1+0}$$

$$= \sqrt{9-0} = 3$$

21.

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)=\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x}=\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}\right)^2-(3x)^2}{\sqrt{9x^2+x}+3x}$$

$$=\lim_{x\to\infty}\frac{\left(9x^2+x\right)-9x^2}{\sqrt{9x^2+x}+3x}=\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}\cdot\frac{1/x}{1/x}$$

$$=\lim_{x\to\infty}\frac{x/x}{\sqrt{9x^2/x^2+x/x^2}+3x/x}=\lim_{x\to\infty}\frac{1}{\sqrt{9+1/x}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{3+3}=\frac{1}{6}$$

31.

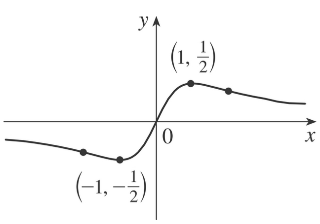
$$\lim_{x\to\infty}\frac{x+x^3+x^5}{1-x^2+x^4}=\lim_{x\to\infty}\frac{(x+x^3+x^5)/x^4}{(1-x^2+x^4)/x^4}\quad\text{[divide by the highest power of } x \text{ in the denominator]}$$

$$=\lim_{x\to\infty}\frac{1/x^3+1/x+x}{1/x^4-1/x^2+1}=\infty$$

because $(1/x^3+1/x+x)\to\infty$ and $(1/x^4-1/x^2+1)\to 1$ as $x\to\infty$.

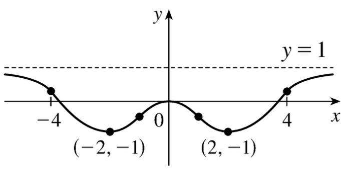
45. $\displaystyle\lim_{x\to\pm\infty}\frac{x}{x^2+1}=\lim_{x\to\pm\infty}\frac{1/x}{1+1/x^2}=\frac{0}{1+0}=0$, so $y=0$ is a horizontal asymptote.

$\displaystyle y'=\frac{x^2+1-x(2x)}{\left(x^2+1\right)^2}=\frac{1-x^2}{\left(x^2+1\right)^2}=0$ when $x=\pm 1$ and $y'>0\Leftrightarrow x^2<1\Leftrightarrow -1<x<1$, so y is increasing on $(-1,1)$ and decreasing on $(-\infty,-1)$ and $(1,\infty)$.



$\displaystyle y''=\frac{\left(1+x^2\right)^2(-2x)-\left(1-x^2\right)2\left(x^2+1\right)2x}{\left(1+x^2\right)^4}=\frac{2x\left(x^2-3\right)}{\left(1+x^2\right)^3}>0\Leftrightarrow x>\sqrt{3}$ or $-\sqrt{3}<x<0$, so y is CU on $\left(\sqrt{3},\infty\right)$ and $\left(-\sqrt{3},0\right)$ and CD on $\left(-\infty,-\sqrt{3}\right)$ and $\left(0,\sqrt{3}\right)$.

51. First we plot the points which are known to be on the graph: $(2,-1)$ and $(0,0)$. We can also draw a short line segment of slope 0 at $x=2$, since we are given that $f'(2)=0$. Now we know that $f'(x)<0$ (that is, the function is decreasing) on $(0,2)$, and that $f''(x)<0$ on $(0,1)$ and $f''(x)>0$ on $(1,2)$. So we must join the points $(0,0)$ and $(2,-1)$ in



such a way that the curve is concave down on $(0,1)$ and concave up on $(1,2)$. The curve must be concave up and increasing on $(2,4)$ and concave down and increasing toward $y=1$ on $(4,\infty)$. Now we just need to reflect the curve in the y-axis, since we are given that f is an even function [the condition that $f(-x)=f(x)$ for all x].