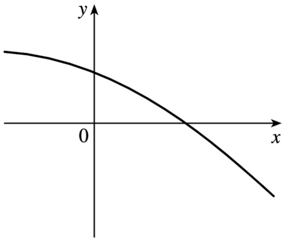
1. **(a)** f is increasing on $(0,6)$ and $(8,9)$.
(b) f is decreasing on $(6,8)$.
(c) f is concave upward on $(2,4)$ and $(7,9)$.
(d) f is concave downward on $(0,2)$ and $(4,7)$.
(e) The points of inflection are $(2,3)$, $(4,4.5)$ and $(7,4)$ (where the concavity changes).

3. **(a)** Use the Increasing/Decreasing (I/D) Test.
(b) Use the Concavity Test.
(c) At any value of x where the concavity changes, we have an inflection point at $(x,f(x))$.

6. **(a)** $f'(x)>0$ and f is increasing on $(0,1)$ and $(3,5)$. $f'(x)<0$ and f is decreasing on $(1,3)$ and $(5,6)$.
(b) Since $f'(x)=0$ at $x=1$ and $x=5$ and f' changes from positive to negative at both values, f changes from increasing to decreasing and has local maxima at $x=1$ and $x=5$. Since $f'(x)=0$ at $x=3$ and f' changes from negative to positive there, f changes from decreasing to increasing and has a local minimum at $x=3$.

7. There is an inflection point at $x=1$ because $f''(x)$ changes from negative to positive there, and so the graph of f changes from concave downward to concave upward. There is an inflection point at $x=7$ because $f''(x)$ changes from positive to negative there, and so the graph of f changes from concave upward to concave downward.

9. The function must be always decreasing and concave downward.



14. **(a)** $f(x)=\dfrac{x^2}{x^2+3} \Rightarrow f'(x)=\dfrac{\left(x^2+3\right)(2x)-x^2(2x)}{\left(x^2+3\right)^2}=\dfrac{6x}{\left(x^2+3\right)^2}$. The denominator is positive so the sign of $f'(x)$ is determined by the sign of x. Thus, $f'(x)>0 \Leftrightarrow x>0$ and $f'(x)<0 \Leftrightarrow x<0$. So f is increasing on $(0,\infty)$ and f is decreasing on $(-\infty,0)$.
(b) f changes from decreasing to increasing at $x=0$. Thus, $f(0)=0$ is a local minimum value.
(c)

$$f^{\prime\prime}(x)=\frac{x^2+3\ ^2(6)-6x\cdot 2\left(x^2+3\right)(2x)}{\left[\left(x^2+3\right)^2\right]^2}=\frac{6\left(x^2+3\right)\left[x^2+3-4x^2\right]}{\left(x^2+3\right)^4}$$

$$=\frac{6\left(3-3x^2\right)}{\left(x^2+3\right)^3}=\frac{-18(x+1)(x-1)}{\left(x^2+3\right)^3}\ .$$

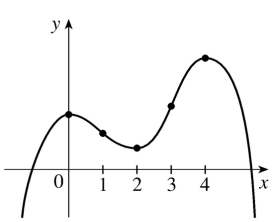
$f^{\prime\prime}(x)>0\Leftrightarrow -1<x<1$ and $f^{\prime\prime}(x)<0\Leftrightarrow x<-1$ or $x>1$. Thus, f is concave upward on $(-1,1)$ and concave downward on $(-\infty,-1)$ and $(1,\infty)$. There are inflection points at $\left(\pm 1,\frac{1}{4}\right)$.

15. **(a)** $f(x)=x-2\sin x$ on $(0,3\pi)\Rightarrow f^{\prime}(x)=1-2\cos x$. $f^{\prime}(x)>0\Leftrightarrow 1-2\cos x>0\Leftrightarrow \cos x<\frac{1}{2}\Leftrightarrow \frac{\pi}{3}<x<\frac{5\pi}{3}$ or $\frac{7\pi}{3}<x<3\pi$. $f^{\prime}(x)<0\Leftrightarrow \cos x>\frac{1}{2}\Leftrightarrow 0<x<\frac{\pi}{3}$ or $\frac{5\pi}{3}<x<\frac{7\pi}{3}$. So f is increasing on $\left(\frac{\pi}{3},\frac{5\pi}{3}\right)$ and $\left(\frac{7\pi}{3},3\pi\right)$, and f is decreasing on $\left(0,\frac{\pi}{3}\right)$ and $\left(\frac{5\pi}{3},\frac{7\pi}{3}\right)$.

(b) f changes from increasing to decreasing at $x=\frac{5\pi}{3}$, and from decreasing to increasing at $x=\frac{\pi}{3}$ and at $x=\frac{7\pi}{3}$. Thus, $f\left(\frac{5\pi}{3}\right)=\frac{5\pi}{3}+\sqrt{3}\approx 6.97$ is a local maximum value and $f\left(\frac{\pi}{3}\right)=\frac{\pi}{3}-\sqrt{3}\approx -0.68$ and $f\left(\frac{7\pi}{3}\right)=\frac{7\pi}{3}-\sqrt{3}\approx 5.60$ are local minimum values.

(c) $f^{\prime\prime}(x)=2\sin x>0\Leftrightarrow 0<x<\pi$ and $2\pi<x<3\pi$, $f^{\prime\prime}(x)<0\Leftrightarrow \pi<x<2\pi$. Thus, f is concave upward on $(0,\pi)$ and $(2\pi,3\pi)$, and f is concave downward on $(\pi,2\pi)$. There are inflection points at (π,π) and $(2\pi,2\pi)$.

23. $f^{\prime}(0)=f^{\prime}(2)=f^{\prime}(4)=0\Leftrightarrow$ horizontal tangents at $x=0$, 2 , 4. $f^{\prime}(x)>0$ if $x<0$ or $2<x<4\Rightarrow f$ is increasing on $(-\infty,0)$ and $(2,4)$. $f^{\prime}(x)<0$ if $0<x<2$ or $x>4\Rightarrow f$ is decreasing on $(0,2)$ and $(4,\infty)$. $f^{\prime\prime}(x)>0$ if $1<x<3\Rightarrow f$ is concave upward on $(1,3)$. $f^{\prime\prime}(x)<0$ if $x<1$ or $x>3\Rightarrow f$ is concave downward on $(-\infty,1)$ and $(3,\infty)$. There are inflection points when $x=1$ and 3.

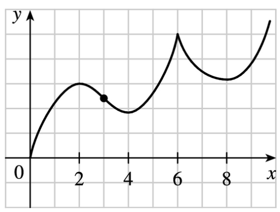


27. **(a)** f is increasing where f' is positive, that is, on $(0,2)$, $(4,6)$, and $(8,\infty)$; and decreasing where f' is negative, that is, on $(2,4)$ and $(6,8)$.

(b) f has local maxima where f' changes from positive to negative, at $x=2$ and at $x=6$, and local minima where f' changes from negative to positive, at $x=4$ and at $x=8$.

(c) f is concave upward (CU) where f' is increasing, that is, on $(3,6)$ and $(6,\infty)$, and concave downward (CD) where f' is decreasing, that is, on $(0,3)$.

(d) There is a point of in.ection where f changes from being CD to being CU, that is, at $x = 3$.

(e) 

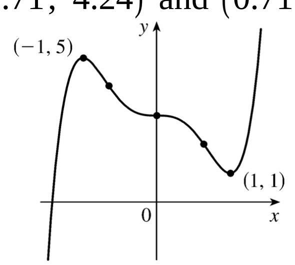
33. **(a)** $h(x)=3x^5-5x^3+3 \Rightarrow h'(x)=15x^4-15x^2=15x^2\left(x^2-1\right)=0$ when $x=0$, ± 1. Since $15x^2$ is nonnegative, $h'(x)>0 \Leftrightarrow x^2>1 \Leftrightarrow |x|>1 \Leftrightarrow x>1$ or $x<-1$, so h is increasing on $(-\infty,-1)$ and $(1,\infty)$ and decreasing on $(-1,1)$, with a horizontal tangent at $x=0$.

(b) Local maximum value $h(-1)=5$, local minimum value $h(1)=1$

(c)

$$\begin{aligned} h''(x) &= 60x^3-30x=30x\left(2x^2-1\right) \\ &= 60x\left(x+\frac{1}{\sqrt{2}}\right)\left(x-\frac{1}{\sqrt{2}}\right) \Rightarrow \end{aligned}$$

$h''(x)>0$ when $x>\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}<x<0$, so h is CU on $\left(-\frac{1}{\sqrt{2}},0\right)$ and $\left(\frac{1}{\sqrt{2}},\infty\right)$ and CD on $\left(-\infty,-\frac{1}{\sqrt{2}}\right)$ and $\left(0,\frac{1}{\sqrt{2}}\right)$. Inflection points at $(0,3)$ and $\left(\pm\frac{1}{\sqrt{2}},3\mp\frac{7}{8}\sqrt{2}\right)$ [about $(-0.71,\ 4.24)$ and $(0.71,\ 1.76)$].

(d) 

35. **(a)** $A(x)=x\sqrt{x+3} \Rightarrow A'(x)=x\cdot \frac{1}{2}(x+3)^{-1/2}+\sqrt{x+3}\cdot 1=\frac{x}{2\sqrt{x+3}}+\sqrt{x+3}=\frac{x+2(x+3)}{2\sqrt{x+3}}=\frac{3x+6}{2\sqrt{x+3}}$.

The domain of A is $[-3,\infty)$. $A'(x)>0$ for $x>-2$ and $A'(x)<0$ for $-3<x<-2$, so A is increasing on $(-2,\infty)$ and decreasing on $(-3,-2)$.

(b) $A(-2)=-2$ is a local minimum value.

(c) $A''(x)=\dfrac{2\sqrt{x+3}\cdot 3-(3x+6)\cdot \frac{1}{\sqrt{x+3}}}{\left(2\sqrt{x+3}\right)^2}=\dfrac{6(x+3)-(3x+6)}{4(x+3)^{3/2}}=\dfrac{3x+12}{4(x+3)^{3/2}}=\dfrac{3(x+4)}{4(x+3)^{3/2}}$ $A''(x)>0$ for all $x>-3$, so A is concave upward on $(-3,\infty)$. There is no inflection point.

(d)