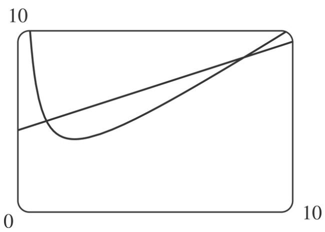
2. $f(x)=x^3-3x^2+2x+5$, $[0,2]$. f is continuous on $[0,2]$ and differentiable on $(0,2)$. Also, $f(0)=5=f(2)$. $f'(c)=0 \Leftrightarrow 3c^2-6c+2=0 \Leftrightarrow c=\dfrac{6\pm\sqrt{36-24}}{6}=1\pm\dfrac{1}{3}\sqrt{3}$, both in $(0,2)$.

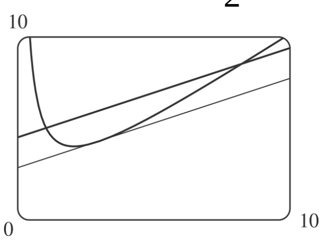
9.

(a), (b) The equation of the secant line is $y-5=\dfrac{8.5-5}{8-1}(x-1) \Leftrightarrow y=\dfrac{1}{2}x+\dfrac{9}{2}$.



(c) $f(x)=x+4/x \Rightarrow f'(x)=1-4/x^2$.

So $f'(c)=\dfrac{1}{2} \Rightarrow c^2=8 \Rightarrow c=2\sqrt{2}$, and $f(c)=2\sqrt{2}+\dfrac{4}{2\sqrt{2}}=3\sqrt{2}$. Thus, an equation of the tangent line is $y-3\sqrt{2}=\dfrac{1}{2}\left(x-2\sqrt{2}\right) \Leftrightarrow y=\dfrac{1}{2}x+2\sqrt{2}$.



11. $f(x)=3x^2+2x+5$, $[-1,1]$. f is continuous on $[-1,1]$ and differentiable on $(-1,1)$ since polynomials are continuous and differentiable on R. $f'(c)=\dfrac{f(b)-f(a)}{b-a} \Leftrightarrow 6c+2=\dfrac{f(1)-f(-1)}{1-(-1)}=\dfrac{10-6}{2}=2 \Leftrightarrow 6c=0 \Leftrightarrow$ $c=0$, which is in $(-1,1)$.

14. $f(x)=\dfrac{x}{x+2}$, $[1,4]$. f is continuous on $[1,4]$ and differentiable on $(1,4)$. $f'(c)=\dfrac{f(b)-f(a)}{b-a} \Leftrightarrow$

$\dfrac{2}{(c+2)^2}=\dfrac{\frac{2}{3}-\frac{1}{3}}{4-1} \Leftrightarrow (c+2)^2=18 \Leftrightarrow c=-2\pm 3\sqrt{2}$. $-2+3\sqrt{2}\approx 2.24$ is in $(1,4)$.

17. Let $f(x)=1+2x+x^3+4x^5$. Then $f(-1)=-6<0$ and $f(0)=1>0$. Since f is a polynomial, it is continuous, so the Intermediate Value Theorem says that there is a number c between -1 and 0 such that $f(c)=0$. Thus, the given equation has a real root. Suppose the equation has distinct real roots a and b with $a<b$. Then $f(a)=f(b)=0$. Since f is a polynomial, it is differentiable on (a,b) and continuous on $[a,b]$. By Rolle's Theorem, there is a number r in (a,b) such that $f'(r)=0$. But $f'(x)=2+3x^2+20x^4 \geq 2$ for all x , so $f'(x)$ can never be 0. This contradiction shows that the equation can't have two distinct real roots. Hence, it has exactly one real root.