

$$3. f(x)=1-x^3+5x^5-3x^7 \Rightarrow F(x)=x-\frac{x^{3+1}}{3+1}+5\frac{x^{5+1}}{5+1}-3\frac{x^{7+1}}{7+1}+C=x-\frac{1}{4}x^4+\frac{5}{6}x^6-\frac{3}{8}x^8+C$$

$$5. f(x)=5x^{1/4}-7x^{3/4} \Rightarrow F(x)=5\frac{x^{1/4+1}}{\frac{1}{4}+1}-7\frac{x^{3/4+1}}{\frac{3}{4}+1}+C=5\frac{x^{5/4}}{5/4}-7\frac{x^{7/4}}{7/4}+C=4x^{5/4}-4x^{7/4}+C$$

$$13. h(x)=x^3+5\sin x \Rightarrow H(x)=\frac{1}{4}x^4+5(-\cos x)+C=\frac{1}{4}x^4-5\cos x+C$$

$$15. f(t)=4\sqrt{t}-\sec t \tan t \Rightarrow F(t)=\frac{4}{3/2}t^{3/2}-\sec t+C=\frac{8}{3}t^{3/2}-\sec t+C_n \text{ on the interval } \left(n\pi-\frac{\pi}{2}, n\pi+\frac{\pi}{2}\right).$$

$$17. f''(x)=6x+12x^2 \Rightarrow f'(x)=6\cdot\frac{x^2}{2}+12\cdot\frac{x^3}{3}+C=3x^2+4x^3+C \Rightarrow$$

$$f(x)=3\cdot\frac{x^3}{3}+4\cdot\frac{x^4}{4}+Cx+D=x^3+x^4+Cx+D \quad [C \text{ and } D \text{ are just arbitrary constants}]$$

$$23. f'(x)=1-6x \Rightarrow f(x)=x-3x^2+C. f(0)=C \text{ and } f(0)=8 \Rightarrow C=8, \text{ so } f(x)=x-3x^2+8.$$

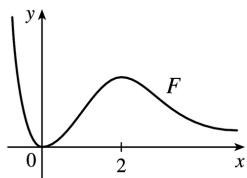
$$29. f''(x)=24x^2+2x+10 \Rightarrow f'(x)=8x^3+x^2+10x+C. f'(1)=8+1+10+C \text{ and } f'(1)=-3 \Rightarrow 19+C=-3 \Rightarrow C=-22, \text{ so } f'(x)=8x^3+x^2+10x-22 \text{ and hence, } f(x)=2x^4+\frac{1}{3}x^3+5x^2-22x+D. f(1)=2+\frac{1}{3}+5-22+D \text{ and } f(1)=5 \Rightarrow D=22-\frac{7}{3}=\frac{59}{3}, \text{ so } f(x)=2x^4+\frac{1}{3}x^3+5x^2-22x+\frac{59}{3}.$$

$$31. f''(\theta)=\sin \theta+\cos \theta \Rightarrow f'(\theta)=-\cos \theta+\sin \theta+C. f'(0)=-1+C \text{ and } f'(0)=4 \Rightarrow C=5, \text{ so } f'(\theta)=-\cos \theta+\sin \theta+5 \text{ and hence, } f(\theta)=-\sin \theta-\cos \theta+5\theta+D. f(0)=-1+D \text{ and } f(0)=3 \Rightarrow D=4, \text{ so } f(\theta)=-\sin \theta-\cos \theta+5\theta+4.$$

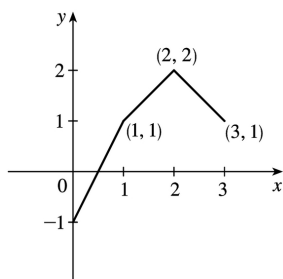
$$35. f''(x)=2+\cos x \Rightarrow f'(x)=2x+\sin x+C \Rightarrow f(x)=x^2-\cos x+Cx+D. f(0)=-1+D \text{ and } f(0)=-1 \Rightarrow D=0. f\left(\frac{\pi}{2}\right)=\pi^2/4+\left(\frac{\pi}{2}\right)C \text{ and } f\left(\frac{\pi}{2}\right)=0 \Rightarrow \left(\frac{\pi}{2}\right)C=-\pi^2/4 \Rightarrow C=-\frac{\pi}{2}, \text{ so } f(x)=x^2-\cos x-\left(\frac{\pi}{2}\right)x.$$

39. b is the antiderivative of f . For small x , f is negative, so the graph of its antiderivative must be decreasing. But both a and c are increasing for small x , so only b can be f 's antiderivative. Also, f is positive where b is increasing, which supports our conclusion.

41. The graph of F will have a minimum at 0 and a maximum at 2, since $f=F'$ goes from negative to positive at $x=0$, and from positive to negative at $x=2$.



43.



$$f'(x) = \begin{cases} 2 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 < x < 2 \\ -1 & \text{if } 2 < x \leq 3 \end{cases} \Rightarrow f(x) = \begin{cases} 2x+C & \text{if } 0 \leq x < 1 \\ x+D & \text{if } 1 < x < 2 \\ -x+E & \text{if } 2 < x \leq 3 \end{cases} \quad f(0) = -1 \Rightarrow 2(0)+C = -1 \Rightarrow C = -1.$$

Starting at the point $(0, -1)$ and moving to the right on a line with slope 2 gets us to the point $(1, 1)$. The slope for $1 < x < 2$ is 1, so we get to the point $(2, 2)$. Here we have used the fact that f is continuous. We can include the point $x=1$ on either the first or the second part of f . The line connecting $(1, 1)$ to $(2, 2)$ is $y=x$, so $D=0$. The slope for $2 < x \leq 3$ is -1 , so we get to $(3, 1)$. $f(3) = 1 \Rightarrow -3+E = 1 \Rightarrow E = 4$. Thus,

$$f(x) = \begin{cases} 2x-1 & \text{if } 0 \leq x \leq 1 \\ x & \text{if } 1 < x < 2 \\ -x+4 & \text{if } 2 \leq x \leq 3 \end{cases}$$

Note that $f'(x)$ does not exist at $x=1$ or at $x=2$.

$$55. a(t) = v'(t) = t - 2 \Rightarrow v(t) = \frac{1}{2}t^2 - 2t + C. \quad v(0) = C \quad \text{and} \quad v(0) = 3 \Rightarrow C = 3, \quad \text{so} \quad v(t) = \frac{1}{2}t^2 - 2t + 3 \quad \text{and}$$

$$s(t) = \frac{1}{6}t^3 - t^2 + 3t + D. \quad s(0) = D \quad \text{and} \quad s(0) = 1 \Rightarrow D = 1, \quad \text{and} \quad s(t) = \frac{1}{6}t^3 - t^2 + 3t + 1.$$

63. Using Exercise with $a=-32$, $v_0=0$, and $s_0=h$ (the height of the cliff), we know that the height at time t is $s(t)=-16t^2+h$. $v(t)=s'(t)=-32t$ and $v(t)=-120 \Rightarrow -32t=-120 \Rightarrow t=3.75$, so $0=s(3.75)=-16(3.75)^2+h \Rightarrow h=16(3.75)^2=225$ ft.

68. $v'(t)=a(t)=-22$. The initial velocity is 50 mi/h $= \frac{50 \cdot 5280}{3600} = \frac{220}{3}$ ft/s, so $v(t)=-22t + \frac{220}{3}$. The car stops when $v(t)=0 \Leftrightarrow t = \frac{220}{3 \cdot 22} = \frac{10}{3}$. Since $s(t)=-11t^2 + \frac{220}{3}t$, the distance covered is $s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3} \cdot \frac{10}{3} = \frac{1100}{9} = 122.\bar{2}$ ft.

69. $a(t)=k$, the initial velocity is 30 mi/h $= 30 \cdot \frac{5280}{3600} = 44$ ft/s, and the final velocity (after 5 seconds) is 50 mi/h $= 50 \cdot \frac{5280}{3600} = \frac{220}{3}$ ft/s. So $v(t)=kt+C$ and $v(0)=44 \Rightarrow C=44$. Thus, $v(t)=kt+44 \Rightarrow v(5)=5k+44$. But $v(5)=\frac{220}{3}$, so $5k+44=\frac{220}{3} \Rightarrow 5k=\frac{88}{3} \Rightarrow k=\frac{88}{15} \approx 5.87$ ft/s².