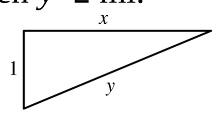
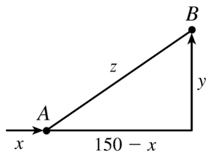
1. $V = x^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = 3x^2\frac{dx}{dt}$

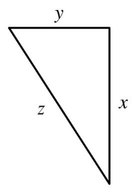
3. $y = x^3 + 2x \Rightarrow \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (3x^2+2)(5) = 5(3x^2+2)$. When $x=2$, $\frac{dy}{dt} = 5(14) = 70$.

7. **(a)** Given: a plane flying horizontally at an altitude of 1 mi and a speed of 500 mi / h passes directly over a radar station. If we let t be time (in hours) and x be the horizontal distance traveled by the plane (in mi), then we are given that $dx/dt = 500$ mi / h.
(b) Unknown: the rate at which the distance from the plane to the station is increasing when it is 2 mi from the station. If we let y be the distance from the plane to the station, then we want to find dy/dt when $y = 2$ mi.
(c) 

(d) By the Pythagorean Theorem, $y^2 = x^2 + 1 \Rightarrow 2y(dy/dt) = 2x(dx/dt)$.
(e) $\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{x}{y}(500)$. Since $y^2 = x^2 + 1$, when $y = 2$, $x = \sqrt{3}$, so $\frac{dy}{dt} = \frac{\sqrt{3}}{2}(500) = 250\sqrt{3} \approx 433$ mi / h.

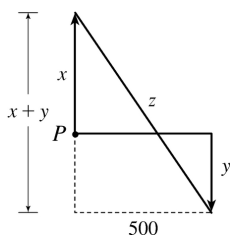
10. **(a)** Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km / h, and ship B is sailing north at 25 km / h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km / h and $dy/dt = 25$ km / h.
(b) Unknown: the rate at which the distance between the ships is changing at 4:00 P.M. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.
(c) 

(d) $z^2 = (150-x)^2 + y^2 \Rightarrow 2z\frac{dz}{dt} = 2(150-x)\left(-\frac{dx}{dt}\right) + 2y\frac{dy}{dt}$
(e) At 4:00 P.M., $x = 4(35) = 140$ and $y = 4(25) = 100 \Rightarrow z = \sqrt{(150-140)^2 + 100^2} = \sqrt{10{,}100}$. So
$\frac{dz}{dt} = \frac{1}{z}\left[(x-150)\frac{dx}{dt} + y\frac{dy}{dt}\right] = \frac{-10(35)+100(25)}{\sqrt{10{,}100}} = \frac{215}{\sqrt{101}} \approx 21.4$ km / h.

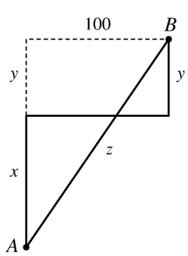
11. 

We are given that $\frac{dx}{dt} = 60$ mi / h and $\frac{dy}{dt} = 25$ mi / h. $z^2 = x^2 + y^2 \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow$
$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$.

After 2 hours, $x = 2(60) = 120$ and $y = 2(25) = 50 \Rightarrow z = \sqrt{120^2 + 50^2} = 130$, so
$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) = \frac{120(60) + 50(25)}{130} = 65$ mi / h.

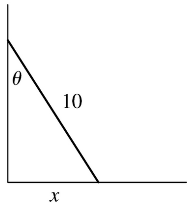
13. 

We are given that $\frac{dx}{dt} = 4$ ft / s and $\frac{dy}{dt} = 5$ ft / s. $z^2 = (x+y)^2 + 500^2 \Rightarrow 2z\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$. 15 minutes after the woman starts, we have $x = (4\text{ft/s})(20\text{min})(60\text{s/min}) = 4800$ ft and $y = 5 \cdot 15 \cdot 60 = 4500 \Rightarrow$
$z = \sqrt{(4800+4500)^2 + 500^2} = \sqrt{86{,}740{,}000}$, so
$\frac{dz}{dt} = \frac{x+y}{z}\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = \frac{4800+4500}{\sqrt{86{,}740{,}000}}(4+5) = \frac{837}{\sqrt{8674}} \approx 8.99$ ft / s .

17. 

We are given that $\frac{dx}{dt} = 35$ km / h and $\frac{dy}{dt} = 25$ km / h. $z^2 = (x+y)^2 + 100^2 \Rightarrow 2z\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$.

At 4:00 P.M., $x = 4(35) = 140$ and $y = 4(25) = 100 \Rightarrow z = \sqrt{(140+100)^2 + 100^2} = \sqrt{67{,}600} = 260$, so
$\frac{dz}{dt} = \frac{x+y}{z}\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = \frac{140+100}{260}(35+25) = \frac{720}{13} \approx 55.4$ km / h.

31. 

We are given that $\frac{dx}{dt} = 2$ ft / s. $\sin\theta = \frac{x}{10} \Rightarrow x = 10\sin\theta \Rightarrow \frac{dx}{dt} = 10\cos\theta \, \frac{d\theta}{dt}$. When $\theta = \frac{\pi}{4}$, $2 = 10\cos\frac{\pi}{4}\,\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{2}{10(1/\sqrt{2})} = \frac{\sqrt{2}}{5}$ rad / s.