

3. We can immediately see that a is the graph of the acceleration function, since at the points where a has a horizontal tangent, neither c nor b is equal to 0. Next, we note that $a=0$ at the point where b has a horizontal tangent, so b must be the graph of the velocity function, and hence, $b' = a$. We conclude that c is the graph of the position function.

$$5. f(x)=x^5+6x^2-7x \Rightarrow f'(x)=5x^4+12x-7 \Rightarrow f''(x)=20x^3+12$$

$$8. y=\theta \sin \theta \Rightarrow y'=\theta \cos \theta +\sin \theta \Rightarrow y''=\theta (-\sin \theta)+\cos \theta \cdot 1+\cos \theta =2\cos \theta -\theta \sin \theta$$

$$9. F(t)=(1-7t)^6 \Rightarrow F'(t)=6(1-7t)^5(-7)=-42(1-7t)^5 \Rightarrow F''(t)=-42 \cdot 5(1-7t)^4(-7)=1470(1-7t)^4$$

$$13. h(x)=\sqrt{x^2+1} \Rightarrow h'(x)=\frac{1}{2}(x^2+1)^{-1/2}(2x)=\frac{x}{\sqrt{x^2+1}} \Rightarrow$$

$$h''(x)=\frac{\sqrt{x^2+1} \cdot 1-x \left[\frac{1}{2}(x^2+1)^{-1/2}(2x) \right]}{\left(\sqrt{x^2+1}\right)^2} = \frac{(x^2+1)^{-1/2}[(x^2+1)-x^2]}{(x^2+1)^1} = \frac{1}{(x^2+1)^{3/2}}$$

$$27. f(\theta)=\cot \theta \Rightarrow f'(\theta)=-\csc^2 \theta \Rightarrow f''(\theta)=-2\csc \theta (-\csc \theta \cdot \cot \theta)=2\csc^2 \theta \cdot \cot \theta \Rightarrow$$

$$f'''(\theta)=2(-2\csc^2 \theta \cdot \cot \theta) \cot \theta +2\csc^2 \theta (-\csc^2 \theta)=-2\csc^2 \theta (2\cot^2 \theta +\csc^2 \theta) \Rightarrow$$

$$f''' \left(\frac{\pi}{6} \right)=-2(2)^2[2(\sqrt{3})^2+(2)^2]=-80$$

$$29. 9x^2+y^2=9 \Rightarrow 18x+2yy'=0 \Rightarrow 2yy'=-18x \Rightarrow y'=-9x/y \Rightarrow$$

$$y''=-9 \left(\frac{y \cdot 1-x \cdot y'}{y^2} \right)=-9 \left(\frac{y-x(-9x/y)}{y^2} \right)=-9 \cdot \frac{y^2+9x^2}{y^3}=-9 \cdot \frac{9}{y^3} \text{ [since } x \text{ and } y \text{ must satisfy the}$$

original equation, $9x^2+y^2=9$]. Thus, $y''=-81/y^3$.

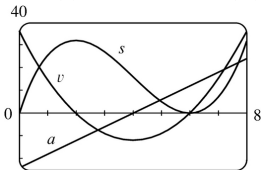
$$35. f(x)=(1+x)^{-1} \Rightarrow f'(x)=-1(1+x)^{-2}, f''(x)=1 \cdot 2(1+x)^{-3}, f^{(3)}(x)=-1 \cdot 2 \cdot 3(1+x)^{-4},$$

$$f^{(4)}(x)=1 \cdot 2 \cdot 3 \cdot 4(1+x)^{-5}, \dots, f^{(n)}(x)=(-1)^n n!(1+x)^{-(n+1)}$$

39. Let $f(x)=\cos x$. Then $Df(2x)=2f'(2x)$, $D^2f(2x)=2^2f''(2x)$, $D^3f(2x)=2^3f'''(2x)$, ..., $D^{(n)}f(2x)=2^n f^{(n)}(2x)$. Since the derivatives of $\cos x$ occur in a cycle of four, and since $103=4(25)+3$, we have $f^{(103)}(x)=f^{(3)}(x)=\sin x$ and $D^{103}\cos 2x=2^{103}f^{(103)}(2x)=2^{103}\sin 2x$.

49. (a) $s=f(t)=t^3-12t^2+36t$, $t \geq 0 \Rightarrow v(t)=f'(t)=3t^2-24t+36$.

$a(t)=v'(t)=6t-24$. $a(3)=6(3)-24=-6$ (m/s) / s or m / s^2 .



(b) -25
 (c) The particle is speeding up when v and a have the same sign. This occurs when $2 < t < 4$ and when $t > 6$. It is slowing down when v and a have opposite signs; that is, when $0 \leq t < 2$ and when $4 < t < 6$.

53. Let $P(x)=ax^2+bx+c$. Then $P'(x)=2ax+b$ and $P''(x)=2a$. $P''(2)=2 \Rightarrow 2a=2 \Rightarrow a=1$.

$P'(2)=3 \Rightarrow 2(1)(2)+b=3 \Rightarrow 4+b=3 \Rightarrow b=-1$.

$P(2)=5 \Rightarrow 1(2)^2+(-1)(2)+c=5 \Rightarrow 2+c=5 \Rightarrow c=3$. So $P(x)=x^2-x+3$.

54. Let $Q(x)=ax^3+bx^2+cx+d$. Then $Q'(x)=3ax^2+2bx+c$, $Q''(x)=6ax+2b$ and $Q'''(x)=6a$. Thus, $Q(1)=a+b+c+d=1$, $Q'(1)=3a+2b+c=3$, $Q''(1)=6a+2b=6$ and $Q'''(1)=6a=12$. Solving these four equations in four unknowns a , b , c and d we get $a=2$, $b=-3$, $c=3$ and $d=-1$, so $Q(x)=2x^3-3x^2+3x-1$.

59.

$$f(x) = g(\sqrt{x}) \Rightarrow f'(x) = g'(\sqrt{x}) \cdot \frac{1}{2} x^{-1/2} = \frac{g'(\sqrt{x})}{2\sqrt{x}} \Rightarrow$$

$$f''(x) = \frac{2\sqrt{x} \cdot g''(\sqrt{x}) \cdot \frac{1}{2} x^{-1/2} - g'(\sqrt{x}) \cdot 2 \cdot \frac{1}{2} x^{-1/2}}{(2\sqrt{x})^2} = \frac{x^{-1/2} [\sqrt{x} g''(\sqrt{x}) - g'(\sqrt{x})]}{4x}$$

$$= \frac{\sqrt{x} g''(\sqrt{x}) - g'(\sqrt{x})}{4x\sqrt{x}}$$