1. **(a)** $\frac{d}{dx}(xy+2x+3x^2)=\frac{d}{dx}(4)\Rightarrow(x\cdot y'+y\cdot 1)+2+6x=0\Rightarrow xy'=-y-2-6x\Rightarrow y'=\frac{-y-2-6x}{x}$ or $y'=-6-\frac{y+2}{x}$.

(b) $xy+2x+3x^2=4\Rightarrow xy=4-2x-3x^2\Rightarrow y=\frac{4-2x-3x^2}{x}=\frac{4}{x}-2-3x$, so $y'=-\frac{4}{x^2}-3$.

(c) From part (a), $y'=\frac{-y-2-6x}{x}=\frac{-(4/x-2-3x)-2-6x}{x}=\frac{-4/x-3x}{x}=-\frac{4}{x^2}-3$.

3. **(a)** $\frac{d}{dx}\left(\frac{1}{x}+\frac{1}{y}\right)=\frac{d}{dx}(1)\Rightarrow-\frac{1}{x^2}-\frac{1}{y^2}y'=0\Rightarrow-\frac{1}{y^2}y'=\frac{1}{x^2}\Rightarrow y'=-\frac{y^2}{x^2}$

(b) $\frac{1}{x}+\frac{1}{y}=1\Rightarrow\frac{1}{y}=1-\frac{1}{x}=\frac{x-1}{x}\Rightarrow y=\frac{x}{x-1}$, so $y'=\frac{(x-1)(1)-(x)(1)}{(x-1)^2}=\frac{-1}{(x-1)^2}$.

(c) $y'=-\frac{y^2}{x^2}=-\frac{[x/(x-1)]^2}{x^2}=-\frac{x^2}{x^2(x-1)^2}=-\frac{1}{(x-1)^2}$

7. $\frac{d}{dx}(x^3+x^2y+4y^2)=\frac{d}{dx}(6)\Rightarrow 3x^2+(x^2y'+y\cdot 2x)+8yy'=0\Rightarrow x^2y'+8yy'=-3x^2-2xy\Rightarrow$
$(x^2+8y)y'=-3x^2-2xy\Rightarrow y'=-\frac{3x^2+2xy}{x^2+8y}=-\frac{x(3x+2y)}{x^2+8y}$

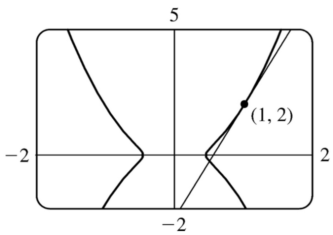
9. $\frac{d}{dx}(x^2y+xy^2)=\frac{d}{dx}(3x)\Rightarrow(x^2y'+y\cdot 2x)+(x\cdot 2yy'+y^2\cdot 1)=3\Rightarrow x^2y'+2xyy'=3-2xy-y^2\Rightarrow$
$y'(x^2+2xy)=3-2xy-y^2\Rightarrow y'=\frac{3-2xy-y^2}{x^2+2xy}$

11. $\frac{d}{dx}(x^2y^2+x\sin y)=\frac{d}{dx}(4)\Rightarrow x^2\cdot 2yy'+y^2\cdot 2x+x\cos y\cdot y'+\sin y\cdot 1=0\Rightarrow$
$2x^2yy'+x\cos y\cdot y'=-2xy^2-\sin y\Rightarrow(2x^2y+x\cos y)y'=-2xy^2-\sin y\Rightarrow y'=\frac{-2xy^2-\sin y}{2x^2y+x\cos y}$

13. $\frac{d}{dx}(4\cos x\sin y)=\frac{d}{dx}(1)\Rightarrow 4[\cos x\cdot\cos y\cdot y'+\sin y\cdot(-\sin x)]=0\Rightarrow$ $y'(4\cos x\cos y)=4\sin x\sin y\Rightarrow y'=\frac{4\sin x\sin y}{4\cos x\cos y}=\tan x\tan y$

21. $\frac{d}{dx}\{1+f(x)+x^2[f(x)]^3\}=\frac{d}{dx}(0)\Rightarrow f'(x)+x^2\cdot 3[f(x)]^2\cdot f'(x)+[f(x)]^3\cdot 2x=0$. If $x=1$, we have $f'(1)+1^2\cdot 3[f(1)]^2\cdot f'(1)+[f(1)]^3\cdot 2(1)=0\Rightarrow f'(1)+1\cdot 3\cdot 2^2\cdot f'(1)+2^3\cdot 2=0\Rightarrow f'(1)+12f'(1)=-16\Rightarrow$ $13f'(1)=-16\Rightarrow f'(1)=-\frac{16}{13}$.

31. **(a)** $y^2=5x^4-x^2\Rightarrow 2yy'=5(4x^3)-2x\Rightarrow y'=\frac{10x^3-x}{y}$. So at the point (1,2) we have $y'=\frac{10(1)^3-1}{2}=\frac{9}{2}$, and an equation of the tangent line is $y-2=\frac{9}{2}(x-1)$ or $y=\frac{9}{2}x-\frac{5}{2}$.

(b) 

35. From Exercise 29, a tangent to the lemniscate will be horizontal if $y'=0\Rightarrow 25x-4x(x^2+y^2)=0\Rightarrow$ $x[25-4(x^2+y^2)]=0\Rightarrow x^2+y^2=\frac{25}{4}$ (**1**). (Note that when x is 0 , y is also 0 , and there is no horizontal tangent at the origin.) Substituting $\frac{25}{4}$ for x^2+y^2 in the equation of the lemniscate, $2(x^2+y^2)^2=25(x^2-y^2)$, we get $x^2-y^2=\frac{25}{8}$ (**2**). Solving (**1**) and (**2**), we have $x^2=\frac{75}{16}$ and $y^2=\frac{25}{16}$, so the four points are $\left(\pm\frac{5\sqrt{3}}{4},\pm\frac{5}{4}\right)$.