1. Let $u=g(x)=4x$ and $y=f(u)=\sin u$. Then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=(\cos u)(4)=4\cos 4x$.

3. Let $u=g(x)=1-x^2$ and $y=f(u)=u^{10}$. Then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=(10u^9)(-2x)=-20x(1-x^2)^9$.

5. Let $u=g(x)=\sin x$ and $y=f(u)=\sqrt{u}$. Then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=\frac{1}{2}u^{-1/2}\cos x=\frac{\cos x}{2\sqrt{u}}=\frac{\cos x}{2\sqrt{\sin x}}$.

6. Let $u=g(x)=\sqrt{x}$ and $y=f(u)=\sin u$. Then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=(\cos u)\left(\frac{1}{2}x^{-1/2}\right)=\frac{\cos u}{2\sqrt{x}}=\frac{\cos\sqrt{x}}{2\sqrt{x}}$.

9. $F(x)=\sqrt[4]{1+2x+x^3}=(1+2x+x^3)^{1/4}\Rightarrow$

$$F'(x)=\frac{1}{4}(1+2x+x^3)^{-3/4}\cdot\frac{d}{dx}(1+2x+x^3)=\frac{1}{4(1+2x+x^3)^{3/4}}\cdot(2+3x^2)$$

$$=\frac{2+3x^2}{4(1+2x+x^3)^{3/4}}=\frac{2+3x^2}{4\sqrt[4]{(1+2x+x^3)^3}}$$

11. $g(t)=\frac{1}{(t^4+1)^3}=(t^4+1)^{-3}\Rightarrow g'(t)=-3(t^4+1)^{-4}(4t^3)=-12t^3(t^4+1)^{-4}=\frac{-12t^3}{(t^4+1)^4}$

19. $y=(2x-5)^4(8x^2-5)^{-3}\Rightarrow$

$$y'=4(2x-5)^3(2)(8x^2-5)^{-3}+(2x-5)^4(-3)(8x^2-5)^{-4}(16x)$$

$$=8(2x-5)^3(8x^2-5)^{-3}-48x(2x-5)^4(8x^2-5)^{-4}$$

[This simplifies to $8(2x-5)^3(8x^2-5)^{-3}(-4x^2+30x-5)$.]

22. $y=x\sin\sqrt{x}\Rightarrow y'=x\cos\sqrt{x}\cdot\frac{1}{2}x^{-1/2}+\sin\sqrt{x}\cdot 1=\frac{1}{2}\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}$

23. $y=\sin(x\cos x)\Rightarrow y'=\cos(x\cos x)\cdot[x(-\sin x)+\cos x\cdot 1]=(\cos x-x\sin x)\cos(x\cos x)$

27. $y=\dfrac{r}{\sqrt{r^2+1}} \Rightarrow$

$$y' = \frac{\sqrt{r^2+1}\,(1)-r\cdot \frac{1}{2}\,(r^2+1)^{-1/2}(2r)}{\left(\sqrt{r^2+1}\right)^2} = \frac{\sqrt{r^2+1}-\dfrac{r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2} = \frac{\dfrac{\sqrt{r^2+1}\,\sqrt{r^2+1}-r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2}$$

$$= \frac{(r^2+1)-r^2}{\left(\sqrt{r^2+1}\right)^3} = \frac{1}{(r^2+1)^{3/2}} \text{ or } (r^2+1)^{-3/2}$$

Another solution: Write y as a product and make use of the Product Rule. $y=r\,(r^2+1)^{-1/2} \Rightarrow$

$$y' = r\cdot -\frac{1}{2}\,(r^2+1)^{-3/2}(2r)+r^2+1)^{-1/2}\cdot 1$$

$$=(r^2+1)^{-3/2}[-r^2+(r^2+1)^1]=(r^2+1)^{-3/2}(1)=(r^2+1)^{-3/2}$$

The step that students usually have trouble with is factoring out $(r^2+1)^{-3/2}$. But this is no different than factoring out x^2 from x^2+x^5 ; that is, we are just factoring out a factor with the *smallest* exponent that appears on it. In this case, $-\dfrac{3}{2}$ is smaller than $-\dfrac{1}{2}$.

29. $y=\tan(\cos x) \Rightarrow y' = \sec^2(\cos x)\cdot(-\sin x) = -\sin x\cdot\sec^2(\cos x)$

33. $y=(1+\cos^2 x)^6 \Rightarrow y' = 6(1+\cos^2 x)^5\cdot 2\cos x(-\sin x) = -12\cos x\sin x(1+\cos^2 x)^5$

39. $y=\sqrt{x+\sqrt{x}} \Rightarrow y' = \dfrac{1}{2}\,(x+\sqrt{x})^{-1/2}\left(1+\dfrac{1}{2}\,x^{-1/2}\right) = \dfrac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\dfrac{1}{2\sqrt{x}}\right)$

41. $y=\sin(\tan\sqrt{\sin x}) \Rightarrow$

$$y' = \cos(\tan\sqrt{\sin x})\cdot\frac{d}{dx}(\tan\sqrt{\sin x}) = \cos(\tan\sqrt{\sin x})\cdot\sec^2\sqrt{\sin x}\cdot\frac{d}{dx}(\sin x)^{1/2}$$

$$=\cos(\tan\sqrt{\sin x})\sec^2\sqrt{\sin x}\cdot\frac{1}{2}\,(\sin x)^{-1/2}\cdot\cos x$$

$$=\cos(\tan\sqrt{\sin x})\left(\sec^2\sqrt{\sin x}\right)\left(\frac{1}{2\sqrt{\sin x}}\right)(\cos x)$$

45. $y=\sin(\sin x) \Rightarrow y' = \cos(\sin x)\cdot\cos x$. At $(\pi,0)$, $y' = \cos(\sin\pi)\cdot\cos\pi = \cos(0)\cdot(-1) = 1(-1) = -1$, and an equation of the tangent line is $y-0=-1(x-\pi)$, or $y=-x+\pi$.

51. For the tangent line to be horizontal, $f'(x)=0$. $f(x)=2\sin x+\sin^2 x \Rightarrow f'(x)=2\cos x+2\sin x\cos x=0$ $\Leftrightarrow 2\cos x(1+\sin x)=0 \Leftrightarrow \cos x=0$ or $\sin x=-1$, so $x=\frac{\pi}{2}+2n\pi$ or $\frac{3\pi}{2}+2n\pi$, where n is any integer. Now $f\left(\frac{\pi}{2}\right)=3$ and $f\left(\frac{3\pi}{2}\right)=-1$, so the points on the curve with a horizontal tangent are $\left(\frac{\pi}{2}+2n\pi,3\right)$ and $\left(\frac{3\pi}{2}+2n\pi,-1\right)$, where n is any integer.

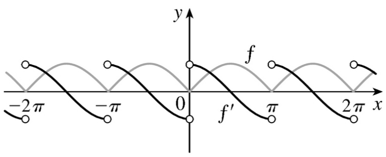
53. $F(x)=f(g(x)) \Rightarrow F'(x)=f'(g(x))\cdot g'(x)$,
so $F'(3)=f'(g(3))\cdot g'(3)=f'(6)\cdot g'(3)=7\cdot 4=28$. Notice that we did not use $f'(3)=2$.

55. **(a)** $h(x)=f(g(x)) \Rightarrow h'(x)=f'(g(x))\cdot g'(x)$, so $h'(1)=f'(g(1))\cdot g'(1)=f'(2)\cdot 6=5\cdot 6=30$.
(b) $H(x)=g(f(x)) \Rightarrow H'(x)=g'(f(x))\cdot f'(x)$, so $H'(1)=g'(f(1))\cdot f'(1)=g'(3)\cdot 4=9\cdot 4=36$.

57. **(a)** $u(x)=f(g(x)) \Rightarrow u'(x)=f'(g(x))g'(x)$. So $u'(1)=f'(g(1))g'(1)=f'(3)g'(1)$. To find $f'(3)$, note that f is linear from (2, 4) to (6, 3), so its slope is $\frac{3-4}{6-2}=-\frac{1}{4}$. To find $g'(1)$, note that g is linear from (0, 6) to (2, 0), so its slope is $\frac{0-6}{2-0}=-3$. Thus, $f'(3)g'(1)=\left(-\frac{1}{4}\right)(-3)=\frac{3}{4}$.
(b) $v(x)=g(f(x)) \Rightarrow v'(x)=g'(f(x))f'(x)$. So $v'(1)=g'(f(1))f'(1)=g'(2)f'(1)$, which does not exist since $g'(2)$ does not exist.
(c) $w(x)=g(g(x)) \Rightarrow w'(x)=g'(g(x))g'(x)$. So $w'(1)=g'(g(1))g'(1)=g'(3)g'(1)$. To find $g'(3)$, note that g is linear from (2, 0) to (5, 2), so its slope is $\frac{2-0}{5-2}=\frac{2}{3}$. Thus, $g'(3)\cdot g'(1)=\left(\frac{2}{3}\right)(-3)=-2$.

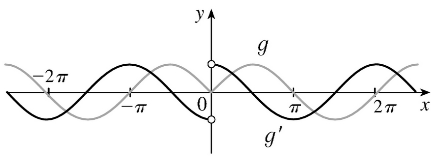
76. **(a)** $f(x)=|x|=\sqrt{x^2}=(x^2)^{1/2} \Rightarrow f'(x)=\frac{1}{2}(x^2)^{-1/2}(2x)=\frac{x}{\sqrt{x^2}}=\frac{x}{|x|}$ for $x\neq 0$.

f is not differentiable at $x=0$.
(b) $f(x)=|\sin x|=\sqrt{\sin^2 x} \Rightarrow f'(x)=\frac{1}{2}(\sin^2 x)^{-1/2}\cdot 2\sin x\cdot\cos x=\frac{\sin x}{|\sin x|}\cos x=\begin{cases}\cos x & \text{if } \sin x>0\\ -\cos x & \text{if } \sin x<0\end{cases}$



f is not differentiable when $x = n\pi$, n an integer.

(c) $g(x) = \sin|x| = \sin\sqrt{x^2} \Rightarrow g'(x) = \cos|x| \cdot \dfrac{x}{|x|} = \dfrac{x}{|x|}\cos x = \begin{cases} \cos x & \text{if } x > 0 \\ -\cos x & \text{if } x < 0 \end{cases}$



g is not differentiable at 0.