6. $g(x)=5x^8-2x^5+6 \Rightarrow g'(x)=5\left(8x^{8-1}\right)-2\left(5x^{5-1}\right)+0=40x^7-10x^4$

9. $V(r)=\frac{4}{3}\pi r^3 \Rightarrow V'(r)=\frac{4}{3}\pi\left(3r^2\right)=4\pi r^2$

15. $y=x^{-2/5} \Rightarrow y'=-\frac{2}{5}x^{(-2/5)-1}=-\frac{2}{5}x^{-7/5}=-\frac{2}{5x^{7/5}}$

20. $u=\sqrt[3]{t^2}+2\sqrt{t^3}=t^{2/3}+2t^{3/2} \Rightarrow u'=\frac{2}{3}t^{-1/3}+2\left(\frac{3}{2}\right)t^{1/2}=\frac{2}{3\sqrt[3]{t}}+3\sqrt{t}$

21. Product Rule:
$y=(x^2+1)(x^3+1) \Rightarrow$

$y'=(x^2+1)(3x^2)+(x^3+1)(2x)=3x^4+3x^2+2x^4+2x=5x^4+3x^2+2x$.
Multiplying first: $y=(x^2+1)(x^3+1)=x^5+x^3+x^2+1 \Rightarrow y'=5x^4+3x^2+2x$ (equivalent).

26. $y=\sqrt{x}\,(x-1)=x^{3/2}-x^{1/2} \Rightarrow y'=\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-1/2}=\frac{1}{2}x^{-1/2}(3x-1)$ [factor out $\frac{1}{2}x^{-1/2}$]
or $y'=\frac{3x-1}{2\sqrt{x}}$.

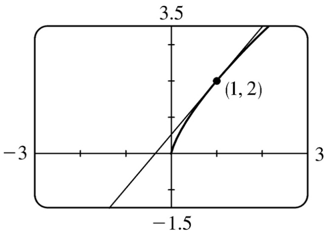
27. $g(x)=\frac{3x-1}{2x+1} \overset{QR}{\Rightarrow} g'(x)=\frac{(2x+1)(3)-(3x-1)(2)}{(2x+1)^2}=\frac{6x+3-6x+2}{(2x+1)^2}=\frac{5}{(2x+1)^2}$

37. $y=\frac{r^2}{1+\sqrt{r}} \Rightarrow$
$$y'=\frac{(1+\sqrt{r})(2r)-r^2\left(\frac{1}{2}r^{-1/2}\right)}{(1+\sqrt{r})^2}=\frac{2r+2r^{3/2}-\frac{1}{2}r^{3/2}}{(1+\sqrt{r})^2}=\frac{2r+\frac{3}{2}r^{3/2}}{(1+\sqrt{r})^2}=\frac{\frac{1}{2}r(4+3r^{1/2})}{(1+\sqrt{r})^2}=\frac{r(4+3\sqrt{r})}{2(1+\sqrt{r})^2}$$

41. $f(x)=\frac{x}{x+c/x} \Rightarrow f'(x)=\frac{(x+c/x)(1)-x(1-c/x^2)}{\left(x+\frac{c}{x}\right)^2}=\frac{x+c/x-x+c/x}{\left(\frac{x^2+c}{x}\right)^2}=\frac{2c/x}{\frac{(x^2+c)^2}{x^2}}\cdot\frac{x^2}{x^2}=\frac{2cx}{(x^2+c)^2}$

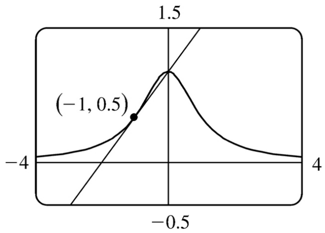
51. $y=\dfrac{2x}{x+1} \Rightarrow y'=\dfrac{(x+1)(2)-(2x)(1)}{(x+1)^2}=\dfrac{2}{(x+1)^2}$. At (1,1) , $y'=\dfrac{1}{2}$, and an equation of the tangent line is $y-1=\dfrac{1}{2}(x-1)$, or $y=\dfrac{1}{2}x+\dfrac{1}{2}$.

53. $y=f(x)=x+\sqrt{x} \Rightarrow f'(x)=1+\dfrac{1}{2}x^{-1/2}$. So the slope of the tangent line at (1,2) is $f'(1)=1+\dfrac{1}{2}(1)=\dfrac{3}{2}$ and its equation is $y-2=\dfrac{3}{2}(x-1)$ or $y=\dfrac{3}{2}x+\dfrac{1}{2}$.



55. **(a)** $y=f(x)=\dfrac{1}{1+x^2} \Rightarrow f'(x)=\dfrac{(1+x^2)(0)-1(2x)}{(1+x^2)^2}=\dfrac{-2x}{(1+x^2)^2}$. So the slope of the tangent line at the point $\left(-1,\dfrac{1}{2}\right)$ is $f'(-1)=\dfrac{2}{2^2}=\dfrac{1}{2}$ and its equation is $y-\dfrac{1}{2}=\dfrac{1}{2}(x+1)$ or $y=\dfrac{1}{2}x+1$.

(b)



57. We are given that $f(5)=1$, $f'(5)=6$, $g(5)=-3$, and $g'(5)=2$.

(a) $(fg)'(5)=f(5)g'(5)+g(5)f'(5)=(1)(2)+(-3)(6)=2-18=-16$

(b) $\left(\dfrac{f}{g}\right)'(5)=\dfrac{g(5)f'(5)-f(5)g'(5)}{[g(5)]^2}=\dfrac{(-3)(6)-(1)(2)}{(-3)^2}=-\dfrac{20}{9}$

(c) $\left(\dfrac{g}{f}\right)'(5)=\dfrac{f(5)g'(5)-g(5)f'(5)}{[f(5)]^2}=\dfrac{(1)(2)-(-3)(6)}{(1)^2}=20$

61. **(a)** From the graphs of f and g, we obtain the following values: $f(1)=2$ since the point (1,2) is on the graph of f; $g(1)=1$ since the point (1,1) is on the graph of g; $f'(1)=2$ since the slope of the line segment between (0,0) and (2,4) is $\frac{4-0}{2-0}=2$; $g'(1)=-1$ since the slope of the line segment between (−2,4) and (2,0) is $\frac{0-4}{2-(-2)}=-1$. Now $u(x)=f(x)g(x)$, so $u'(1)=f(1)g'(1)+g(1)f'(1)=2\cdot(-1)+1\cdot 2=0$.

(b) $v(x)=f(x)/g(x)$, so $v'(5)=\frac{g(5)f'(5)-f(5)g'(5)}{[g(5)]^2}=\frac{2\left(-\frac{1}{3}\right)-3\cdot\frac{2}{3}}{2^2}=\frac{-\frac{8}{3}}{4}=-\frac{2}{3}$

63. **(a)** $y=xg(x)\Rightarrow y'=xg'(x)+g(x)\cdot 1=xg'(x)+g(x)$

(b) $y=\frac{x}{g(x)}\Rightarrow y'=\frac{g(x)\cdot 1-xg'(x)}{[g(x)]^2}=\frac{g(x)-xg'(x)}{[g(x)]^2}$

(c) $y=\frac{g(x)}{x}\Rightarrow y'=\frac{xg'(x)-g(x)\cdot 1}{(x)^2}=\frac{xg'(x)-g(x)}{x^2}$

67. $y=x^3-x^2-x+1$ has a horizontal tangent when $y'=3x^2-2x-1=0$. $(3x+1)(x-1)=0\Leftrightarrow$ $x=1$ or $-\frac{1}{3}$. Therefore, the points are $(1,0)$ and $\left(-\frac{1}{3},\frac{32}{27}\right)$.

71. $y=6x^3+5x-3\Rightarrow m=y'=18x^2+5$, but $x^2\geq 0$ for all x, so $m\geq 5$ for all x.

83. $y=f(x)=ax^2\Rightarrow f'(x)=2ax$. So the slope of the tangent to the parabola at $x=2$ is $m=2a(2)=4a$. The slope of the given line, $2x+y=b\Leftrightarrow y=-2x+b$, is seen to be -2, so we must have $4a=-2\Leftrightarrow a=-\frac{1}{2}$. So when $x=2$, the point in question has y–coordinate $-\frac{1}{2}\cdot 2^2=-2$. Now we simply require that the given line, whose equation is $2x+y=b$, pass through the point $(2,-2)$: \ $2(2)+(-2)=b\Leftrightarrow b=2$. So we must have $a=-\frac{1}{2}$ and $b=2$.