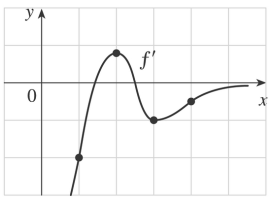
1. *Note:* Your answers may vary depending on your estimates. By estimating the slopes of tangent lines on the graph of f, it appears that

(a) $f'(1) \approx -2$ (b) $f'(2) \approx 0.8$

(c) $f'(3) \approx -1$ (d) $f'(4) \approx -0.5$



4. **(a)** $(a)' =$ II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

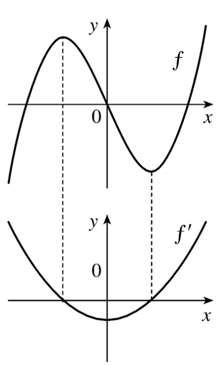
(b) $(b)' =$ IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

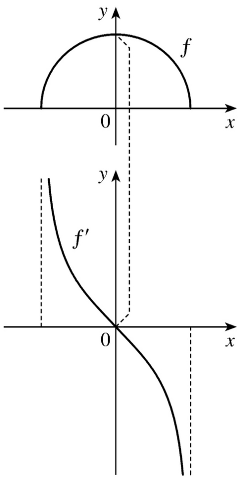
(c) $(c)' =$ I, since the slopes of the tangents to graph (c) are negative for $x<0$ and positive for $x>0$, as are the function values of graph I.

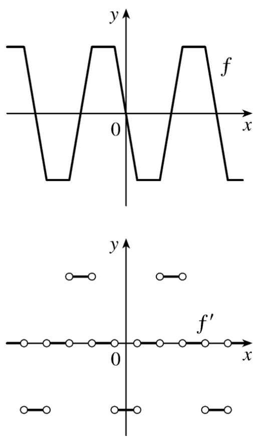
(d) $(d)' =$ III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

Hints for Exercises 5 – 13: First plot x –intercepts on the graph of f' for any horizontal tangents on the graph of f. Look for any corners on the graph of f – there will be a discontinuity on the graph of f'. On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.

5.



10. 

11. 

23.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^3-3(x+h)+5]-(x^3-3x+5)}{h}$$

$$= \lim_{h\to 0} \frac{(x^3+3x^2h+3xh^2+h^3-3x-3h+5)-(x^3-3x+5)}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3-3h}{h} = \lim_{h\to 0} \frac{h(3x^2+3xh+h^2-3)}{h}$$

$$= \lim_{h\to 0}(3x^2+3xh+h^2-3)=3x^2-3$$

Domain of f = domain of f' = R.

25.

$$g'(x)=\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}=\lim_{h\to 0}\frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h}\left[\frac{\sqrt{1+2(x+h)}+\sqrt{1+2x}}{\sqrt{1+2(x+h)}+\sqrt{1+2x}}\right]$$

$$=\lim_{h\to 0}\frac{(1+2x+2h)-(1+2x)}{h[\sqrt{1+2(x+h)}+\sqrt{1+2x}]}=\lim_{h\to 0}\frac{2}{\sqrt{1+2x+2h}+\sqrt{1+2x}}=\frac{2}{2\sqrt{1+2x}}=\frac{1}{\sqrt{1+2x}}$$

Domain of $g=\left[-\frac{1}{2},\infty\right)$, domain of $g'=\left(-\frac{1}{2},\infty\right)$.

27.

$$G'(t)=\lim_{h\to 0}\frac{G(t+h)-G(t)}{h}=\lim_{h\to 0}\frac{\frac{4(t+h)}{(t+h)+1}-\frac{4t}{t+1}}{h}=\lim_{h\to 0}\frac{\frac{4(t+h)(t+1)-4t(t+h+1)}{(t+h+1)(t+1)}}{h}$$

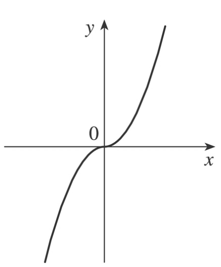
$$=\lim_{h\to 0}\frac{(4t^2+4ht+4t+4h)-(4t^2+4ht+4t)}{h(t+h+1)(t+1)}$$

$$=\lim_{h\to 0}\frac{4h}{h(t+h+1)(t+1)}=\lim_{h\to 0}\frac{4}{(t+h+1)(t+1)}=\frac{4}{(t+1)^2}$$

Domain of G= domain of $G'=(-\infty,-1)\cup(-1,\infty)$.

35. f is not differentiable at $x=-1$ or at $x=11$ because the graph has vertical tangents at those points; at $x=4$, because there is a discontinuity there; and at $x=8$, because the graph has a corner there.

43. **(a)** $f(x)=x|x|=\begin{cases} x^2 & \text{if } x\geq 0 \\ -x^2 & \text{if } x<0 \end{cases}$



(b) Since $f(x)=x^2$ for $x\geq 0$, we have $f'(x)=2x$ for $x>0$. Similarly, since $f(x)=-x^2$ for $x<0$, we have $f'(x)=-2x$ for $x<0$. At $x=0$, we have

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x|x|}{x} = \lim_{x \to 0} |x| = 0.$$

So f is differentiable at 0. Thus, f is differentiable for all x .

(c) From part (b), we have $f'(x) = \left\{ \begin{array}{ll} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{array} \right\} = 2|x|$.