

The line from P(2, f(2)) to Q(2+h, f(2+h)) is the line that has slope $\frac{f(2+h)-f(2)}{h}$

3. g'(0) is the only negative value. The slope at x=4 is smaller than the slope at x=2 and both are smaller than the slope at x=-2. Thus, g'(0)<0<g'(4)<g'(2)<g'(-2).

5.

We begin by drawing a curve through the origin at a slope of 3 to satisfy f(0)=0 and f'(0)=3. Since f'(1)=0, we will round off our figure so that there is a horizontal tangent directly over x=1. Lastly, we make sure that the curve has a slope of -1 as we pass over x=2. Two of the many possibilities are shown.



9. (a) Using Definition 2 with $F(x)=x^3-5x+1$ and the point (1,-3), we have

$$F'(1) = \lim_{h \to 0} \frac{F(1+h) - F(1)}{h} = \lim_{h \to 0} \frac{[(1+h)^3 - 5(1+h) + 1] - (-3)}{h}$$
$$= \lim_{h \to 0} \frac{(1+3h+3h^2 + h^3 - 5 - 5h + 1) + 3}{h} = \lim_{h \to 0} \frac{h^3 + 3h^2 - 2h}{h}$$
$$= \lim_{h \to 0} \frac{h(h^2 + 3h - 2)}{h} = \lim_{h \to 0} (h^2 + 3h - 2) = -2$$

So an equation of the tangent line at (1,-3) is $y-(-3)=-2(x-1) \Leftrightarrow y=-2x-1$. (b)



11. (a)
$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{3^{1+h} - 3^1}{h}$$

So let $F(h) = \frac{3^{1+h} - 3}{h}$. We calculate:

h	F(h)	h	F(h)
0.1	3.484	-0.1	3.121
0.01	3.314	-0.01	3.278
0.001	3.298	-0.001	3.294
0.0001	3.296	-0.0001	3.296

We estimate that
$$f'(1) \approx 3.296$$
.



From the graph, we estimate that the slope of the tangent is about $\frac{3.2-2.8}{1.06-0.94} = \frac{0.4}{0.12} \approx 3.3$.

13. Use Definition 2 with
$$f(x)=3-2x+4x^2$$
.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{[3-2(a+h)+4(a+h)^2]-(3-2a+4a^2)}{h}$$

$$=\lim_{h\to 0}\frac{[3-2a-2h+4a^2+8ah+4h^2]-(3-2a+4a^2)}{h}=\lim_{h\to 0}\frac{-2h+8ah+4h^2}{h}=\lim_{h\to 0}\frac{h(-2+8a+4h)}{h}=\lim_{h\to 0}(-2+8a+4h)=-2+8a$$

19. By Definition 2,
$$\lim_{h \to 0} \frac{(1+h)^{10}-1}{h} = f'(1)$$
, where $f(x) = x^{10}$ and $a = 1$. Or: By Definition 2,
 $\lim_{h \to 0} \frac{(1+h)^{10}-1}{h} = f'(0)$, where $f(x) = (1+x)^{10}$ and $a = 0$.

29. (a) f'(v) is the rate at which the fuel consumption is changing with respect to the speed. Its units are (gal / h) / (mi / h).

(b) The fuel consumption is decreasing by 0.05(gal / h) / (mi / h) as the car's speed reaches 20 mi/h. So if you increase your speed to 21 mi/h, you could expect to decrease your fuel consumption by about 0.05(gal / h) / (mi / h).

31. T'(10) is the rate at which the temperature is changing at 10:00 A.M. To estimate the value of T'(10), we will average the difference quotients obtained using the times t=8 and t=12. Let $A = \frac{T(8) - T(10)}{8 - 10} = \frac{72 - 81}{-2} = 4.5$ and $B = \frac{T(12) - T(10)}{12 - 10} = \frac{88 - 81}{2} = 3.5$. Then $T'(10) = \lim_{t \to 10} \frac{T(t) - T(10)}{t - 10} \approx \frac{A + B}{2} = \frac{4.5 + 3.5}{2} = 4^{\circ} F / h.$