1. 



The line from $P(2, f(2))$ to $Q(2+h, f(2+h))$ is the line that has slope $\frac{f(2+h)-f(2)}{h}$
3. $g^{\prime}(0)$ is the only negative value. The slope at $x=4$ is smaller than the slope at $x=2$ and both are smaller than the slope at $x=-2$. Thus, $g^{\prime}(0)<0<g^{\prime}(4)<g^{\prime}(2)<g^{\prime}(-2)$.
5.

We begin by drawing a curve through the origin at a slope of 3 to satisfy $f(0)=0$ and $f^{\prime}(0)=3$. Since $f^{\prime}(1)=0$, we will round off our figure so that there is a horizontal tangent directly over $x=1$. Lastly, we make sure that the curve has a slope of -1 as we pass over $x=2$. Two of the many possibilities are shown.

9. (a) Using Definition 2 with $F(x)=x^{3}-5 x+1$ and the point $(1,-3)$, we have

$$
\begin{aligned}
F^{\prime}(1) & =\lim _{h \rightarrow 0} \frac{F(1+h)-F(1)}{h}=\lim _{h \rightarrow 0} \frac{\left[(1+h)^{3}-5(1+h)+1\right]-(-3)}{h} \\
& =\lim _{h \rightarrow 0} \frac{\left(1+3 h+3 h^{2}+h^{3}-5-5 h+1\right)+3}{h}=\lim _{h \rightarrow 0} \frac{h^{3}+3 h^{2}-2 h}{h} \\
& =\lim _{h \rightarrow 0} \frac{h\left(h^{2}+3 h-2\right)}{h}=\lim _{h \rightarrow 0}\left(h^{2}+3 h-2\right)=-2
\end{aligned}
$$

So an equation of the tangent line at $(1,-3)$ is $y-(-3)=-2(x-1) \Leftrightarrow y=-2 x-1$.
(b)

11. (a) $f^{\prime}(1)=\lim _{h \rightarrow 0} \frac{f(1+h)-f(1)}{h}=\lim _{h \rightarrow 0} \frac{3^{1+h}-3^{1}}{h}$.

So let $F(h)=\frac{3^{1+h}-3}{h}$. We calculate:

| $h$ | $F(h)$ | $h$ | $F(h)$ |
| :--- | :--- | :--- | :--- |
| 0.1 | 3.484 | -0.1 | 3.121 |
| 0.01 | 3.314 | -0.01 | 3.278 |
| 0.001 | 3.298 | -0.001 | 3.294 |
| 0.0001 | 3.296 | -0.0001 | 3.296 |

We estimate that $f^{\prime}(1) \approx 3.296$.
(b)


From the graph, we estimate that the slope of the tangent is about $\frac{3.2-2.8}{1.06-0.94}=\frac{0.4}{0.12} \approx 3.3$.
13. Use Definition 2 with $f(x)=3-2 x+4 x^{2}$.

$$
\begin{aligned}
& f^{\prime}(a)=\lim _{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}=\lim _{h \rightarrow 0} \frac{\left[3-2(a+h)+4(a+h)^{2}\right]-\left(3-2 a+4 a^{2}\right)}{h} \\
& =\lim _{h \rightarrow 0} \frac{\left[3-2 a-2 h+4 a^{2}+8 a h+4 h^{2}\right]-\left(3-2 a+4 a^{2}\right)}{h}=\lim _{h \rightarrow 0} \frac{-2 h+8 a h+4 h^{2}}{h}=\lim _{h \rightarrow 0} \frac{h(-2+8 a+4 h)}{h}=\lim _{h \rightarrow 0}(-2+8 a+4 h)=-2+8 a
\end{aligned}
$$

19. By Definition 2, $\lim _{h \rightarrow 0} \frac{(1+h)^{10}-1}{h}=f^{\prime}(1)$, where $f(x)=x^{10}$ and $a=1$. Or: By Definition 2,
$\lim _{h \rightarrow 0} \frac{(1+h)^{10}-1}{h}=f^{\prime}(0)$, where $f(x)=(1+x)^{10}$ and $a=0$.
20. (a) $f^{\prime}(v)$ is the rate at which the fuel consumption is changing with respect to the speed. Its units are $(\mathrm{gal} / \mathrm{h}) /(\mathrm{mi} / \mathrm{h})$.
(b) The fuel consumption is decreasing by $0.05(\mathrm{gal} / \mathrm{h}) /(\mathrm{mi} / \mathrm{h})$ as the car's speed reaches $20 \mathrm{mi} / \mathrm{h}$. So if you increase your speed to $21 \mathrm{mi} / \mathrm{h}$, you could expect to decrease your fuel consumption by about $0.05(\mathrm{gal} / \mathrm{h}) /(\mathrm{mi} / \mathrm{h})$.
$31 . T^{\prime}(10)$ is the rate at which the temperature is changing at 10:00 A.M. To estimate the value of $T^{\prime}{ }^{\prime}(10)$, we will average the difference quotients obtained using the times $t=8$ and $t=12$. Let $A=\frac{T(8)-T(10)}{8-10}=\frac{72-81}{-2}=4.5$ and $B=\frac{T(12)-T(10)}{12-10}=\frac{88-81}{2}=3.5$. Then
$T^{\prime}(10)=\lim _{t \rightarrow 10} \frac{T(t)-T(10)}{t-10} \approx \frac{A+B}{2}=\frac{4.5+3.5}{2}=4{ }^{\circ} \mathrm{F} / \mathrm{h}$.
