1. **(a)** This is just the slope of the line through two points: $m_{PQ}=\dfrac{\Delta y}{\Delta x}=\dfrac{f(x)-f(3)}{x-3}$.

(b) This is the limit of the slope of the secant line PQ as Q approaches P : $m=\lim\limits_{x\to 3}\dfrac{f(x)-f(3)}{x-3}$.

3. The slope at D is the largest positive slope, followed by the positive slope at E. The slope at C is zero. The slope at B is steeper than at A (both are negative). In decreasing order, we have the slopes at: D , E , C , A , and B.

5. **(a)**

(i) Using Definition 1,

$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}\ \lim_{x\to -3}\frac{f(x)-f(-3)}{x-(-3)}=\lim_{x\to -3}\frac{\left(x^2+2x\right)-(3)}{x-(-3)}=\lim_{x\to -3}\frac{(x+3)(x-1)}{x+3}$$

$$=\lim_{x\to -3}(x-1)=-4$$

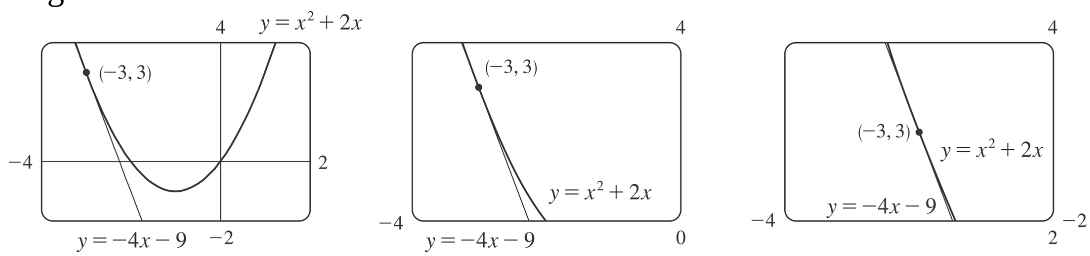
(ii) Using Equation 2,

$$m=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{f(-3+h)-f(-3)}{h}=\lim_{h\to 0}\frac{\left[(-3+h)^2+2(-3+h)\right]-(3)}{h}$$

$$=\lim_{h\to 0}\frac{9-6h+h^2-6+2h-3}{h}=\lim_{h\to 0}\frac{h(h-4)}{h}=\lim_{h\to 0}(h-4)=-4$$

(b) Using the point–slope form of the equation of a line, an equation of the tangent line is $y-3=-4(x+3)$. Solving for y gives us $y=-4x-9$, which is the slope–intercept form of the equation of the tangent line.

(c)



9. Using (1) with $f(x)=\dfrac{x-1}{x-2}$ and $P(3, 2)$,

$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to 3}\frac{\dfrac{x-1}{x-2}-2}{x-3}=\lim_{x\to 3}\frac{\dfrac{x-1-2(x-2)}{x-2}}{x-3}=\lim_{x\to 3}\frac{3-x}{(x-2)(x-3)}$$

$$=\lim_{x\to 3}\frac{-1}{x-2}=\frac{-1}{1}=-1.$$

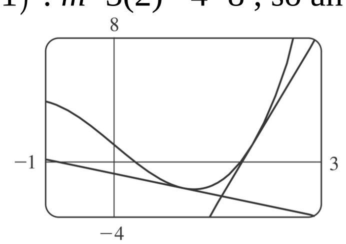
Tangent line: $y-2=-1(x-3)\Leftrightarrow y-2=-x+3\Leftrightarrow y=-x+5$

13. **(a)** Using (1),

$$m=\lim_{x\to a}\frac{\left(x^3-4x+1\right)-\left(a^3-4a+1\right)}{x-a}=\lim_{x\to a}\frac{\left(x^3-a^3\right)-4(x-a)}{x-a}$$

$$=\lim_{x\to a}\frac{(x-a)\left(x^2+ax+a^2\right)-4(x-a)}{x-a}=\lim_{x\to a}\left(x^2+ax+a^2-4\right)=3a^2-4$$

(b) At $(1,-2)$: $m=3(1)^2-4=-1$, so an equation of the tangent line is $y-(-2)=-1(x-1)\Leftrightarrow y=-x-1$. At $(2,1)$: $m=3(2)^2-4=8$, so an equation of the tangent line is $y-1=8(x-2)\Leftrightarrow y=8x-15$.

(c) 

15. **(a)** Since the slope of the tangent at $t=0$ is 0 , the car's initial velocity was 0.
(b) The slope of the tangent is greater at C than at B , so the car was going faster at C .
(c) Near A , the tangent lines are becoming steeper as x increases, so the velocity was increasing, so the car was speeding up. Near B , the tangent lines are becoming less steep, so the car was slowing down. The steepest tangent near C is the one at C , so at C the car had just finished speeding up, and was about to start slowing down.
(d) Between D and E , the slope of the tangent is 0 , so the car did not move during that time.

17. Let $s(t)=40t-16t^2$.

$$v(2)=\lim_{t\to 2}\frac{s(t)-s(2)}{t-2}=\lim_{t\to 2}\frac{(40t-16t^2)-16}{t-2}=\lim_{t\to 2}\frac{-16t^2+40t-16}{t-2}=\lim_{t\to 2}\frac{-8(2t^2-5t+2)}{t-2}$$

$$=\lim_{t\to 2}\frac{-8(t-2)(2t-1)}{t-2}=-8\lim_{t\to 2}(2t-1)=-8(3)=-24$$

Thus, the instantaneous velocity when $t=2$ is -24 ft / s.

20. **(a)** The average velocity between times t and $t+h$ is

$$\frac{s(t+h)-s(t)}{(t+h)-t} = \frac{(t+h)^2-8(t+h)+18-\left(t^2-8t+18\right)}{h}$$

$$= \frac{t^2+2th+h^2-8t-8h+18-t^2+8t-18}{h} = \frac{2th+h^2-8h}{h}$$

$$= (2t+h-8) \text{ m/s}$$

$(i)[3,4]$: $t=3$, $h=4-3=1$, so the average velocity is $2(3)+1-8=-1$ m / s.

$(ii)[3.5,4]$: $t=3.5$, $h=0.5$, so the average velocity is $2(3.5)+0.5-8=-0.5$ m / s.

$(iii)[4,5]$: $t=4$, $h=1$, so the average velocity is $2(4)+1-8=1$ m / s.

$(iv)[4,4.5]$: $t=4$, $h=0.5$, so the average velocity is
$2(4)+0.5-8=0.5$ m / s.

(b) $v(t)=\lim_{h\to 0}\frac{s(t+h)-s(t)}{h}=\lim_{h\to 0}(2t+h-8)=2t-8$, so $v(4)=0$.

(c)