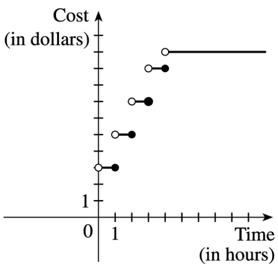
1. From Definition 1, $\lim_{x\to 4} f(x)=f(4)$.

3. **(a)** The following are the numbers at which f is discontinuous and the type of discontinuity at that number: –4 (removable), –2 (jump), 2 (jump), 4 (infinite).
(b) f is continuous from the left at –2 since $\lim_{x\to -2^-} f(x)=f(-2)$. f is continuous from the right at 2 and 4 since $\lim_{x\to 2^+} f(x)=f(2)$ and $\lim_{x\to 4^+} f(x)=f(4)$. It is continuous from neither side at –4 since $f(-4)$ is undefined.

7. **(a)** 

(b) There are discontinuities at times $t=1$, 2 , 3 , and 4. A person parking in the lot would want to keep in mind that the charge will jump at the beginning of each hour.

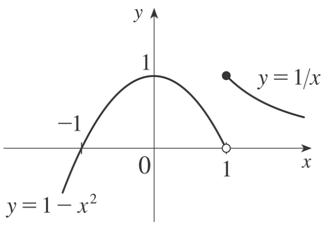
9. Since f and g are continuous functions,

$$\begin{aligned}\lim_{x\to 3}[2f(x)-g(x)] &= 2\lim_{x\to 3} f(x)-\lim_{x\to 3} g(x) \quad \text{[by Limit Laws 2 and 3]}\\ &= 2f(3)-g(3) \quad \text{[by continuity of } f \text{ and } g \text{ at } x=3]\\ &= 2\cdot 5-g(3)=10-g(3)\end{aligned}$$

Since it is given that $\lim_{x\to 3}[2f(x)-g(x)]=4$, we have $10-g(3)=4$, so $g(3)=6$.

17. $f(x)=\begin{cases} 1-x^2 & \text{if} x<1 \\ 1/x & \text{if} x\geq 1 \end{cases}$

The left–hand limit of f at $a=1$ is $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(1-x^2)=0$. The right–hand limit of f at $a=1$ is $\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(1/x)=1$. Since these limits are not equal, $\lim_{x\to 1} f(x)$ does not exist and f is discontinuous at 1.

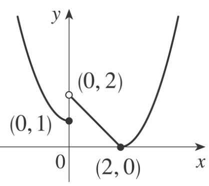


33. $f(x)=\begin{cases} x^2 & \text{if } x<1 \\ \sqrt{x} & \text{if } x\geq 1 \end{cases}$

By Theorem 5, since $f(x)$ equals the polynomial x^2 on $(-\infty,1)$, f is continuous on $(-\infty,1)$. By Theorem 7, since $f(x)$ equals the root function $\sqrt{x}$ on $(1,\infty)$, f is continuous on $(1,\infty)$. At $x=1$, $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-} x^2=1$ and $\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+} \sqrt{x}=1$. Thus, $\lim_{x\to 1} f(x)$ exists and equals 1. Also, $f(1)=\sqrt{1}=1$. Thus, f is continuous at $x=1$. We conclude that f is continuous on $(-\infty,\infty)$.

35. $f(x)=\begin{cases} 1+x^2 & \text{if } x\leq 0 \\ 2-x & \text{if } 0<x\leq 2 \\ (x-2)^2 & \text{if } x>2 \end{cases}$

f is continuous on $(-\infty,0)$, $(0,2)$, and $(2,\infty)$ since it is a polynomial on each of these intervals. Now $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-} (1+x^2)=1$ and



$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} (2-x)=2$, so f is discontinuous at 0. Since $f(0)=1$, f is continuous from the left at 0. Also, $\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-} (2-x)=0$, $\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+} (x-2)^2=0$, and $f(2)=0$, so f is continuous at 2. The only number at which f is discontinuous is 0.

39. f is continuous on $(-\infty,3)$ and $(3,\infty)$. Now $\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(cx+1)=3c+1$ and $\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}\left(cx^2-1\right)=9c-1$. So f is continuous $\Leftrightarrow 3c+1=9c-1 \Leftrightarrow 6c=2 \Leftrightarrow c=\frac{1}{3}$. Thus, for f to be continuous on $(-\infty,\infty)$, $c=\frac{1}{3}$.

43. $f(x)=x^3-x^2+x$ is continuous on the interval $[2,3]$, $f(2)=6$, and $f(3)=21$. Since $6<10<21$, there is a number c in $(2,3)$ such that $f(c)=10$ by the Intermediate Value Theorem.