

1. (a)

$$\begin{aligned}\lim_{x \rightarrow a} [f(x)+h(x)] &= \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} h(x) \\ &= -3 + 8 = 5\end{aligned}$$

$$(b) \lim_{x \rightarrow a} [f(x)]^2 = \left[\lim_{x \rightarrow a} f(x) \right]^2 = (-3)^2 = 9$$

$$(c) \lim_{x \rightarrow a} \sqrt[3]{h(x)} = \sqrt[3]{\lim_{x \rightarrow a} h(x)} = \sqrt[3]{8} = 2$$

$$(d) \lim_{x \rightarrow a} \frac{1}{f(x)} = \frac{1}{\lim_{x \rightarrow a} f(x)} = \frac{1}{-3} = -\frac{1}{3}$$

$$(e) \lim_{x \rightarrow a} \frac{f(x)}{h(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} h(x)} = \frac{-3}{8} = -\frac{3}{8}$$

$$(f) \lim_{x \rightarrow a} \frac{g(x)}{f(x)} = \frac{\lim_{x \rightarrow a} g(x)}{\lim_{x \rightarrow a} f(x)} = \frac{0}{-3} = 0$$

(g) The limit does not exist, since $\lim_{x \rightarrow a} g(x) = 0$ but $\lim_{x \rightarrow a} f(x) \neq 0$.

$$(h) \lim_{x \rightarrow a} \frac{2f(x)}{h(x)-f(x)} = \frac{2\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} h(x) - \lim_{x \rightarrow a} f(x)} = \frac{2(-3)}{8 - (-3)} = -\frac{6}{11}$$

$$2. (a) \lim_{x \rightarrow 2} [f(x)+g(x)] = \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) = 2 + 0 = 2$$

(b) $\lim_{x \rightarrow 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

$$(c) \lim_{x \rightarrow 0} [f(x)g(x)] = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x) = 0 \cdot 1.3 = 0$$

(d) Since $\lim_{x \rightarrow -1} g(x) = 0$ and g is in the denominator, but $\lim_{x \rightarrow -1} f(x) = -1 \neq 0$, the given limit does not exist.

$$(e) \lim_{x \rightarrow 2} x^3 f(x) = \left[\lim_{x \rightarrow 2} x^3 \right] \left[\lim_{x \rightarrow 2} f(x) \right] = 2^3 \cdot 2 = 16$$

$$(f) \lim_{x \rightarrow 1} \sqrt{3+f(x)} = \sqrt{3 + \lim_{x \rightarrow 1} f(x)} = \sqrt{3+1} = 2$$

5.

$$\begin{aligned} \lim_{x \rightarrow 3} (x^2 - 4)(x^3 + 5x - 1) &= \lim_{x \rightarrow 3} (x^2 - 4) \cdot \lim_{x \rightarrow 3} (x^3 + 5x - 1) && \text{[Limit Law 4]} \\ &= \left(\lim_{x \rightarrow 3} x^2 - \lim_{x \rightarrow 3} 4 \right) \cdot \left(\lim_{x \rightarrow 3} x^3 + 5 \lim_{x \rightarrow 3} x - \lim_{x \rightarrow 3} 1 \right) && \text{[2, 1 and 3]} \\ &= (3^2 - 4) \cdot (3^3 + 5 \cdot 3 - 1) && \text{[7, 8 and 9]} \\ &= 5 \cdot 41 = 205 \end{aligned}$$

$$11. \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x \rightarrow 2} (x+3) = 2+3=5$$

$$15. \lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3} = \lim_{t \rightarrow -3} \frac{(t+3)(t-3)}{(2t+1)(t+3)} = \lim_{t \rightarrow -3} \frac{t-3}{2t+1} = \frac{-3-3}{2(-3)+1} = \frac{-6}{-5} = \frac{6}{5}$$

$$17. \lim_{h \rightarrow 0} \frac{(4+h)^2 - 16}{h} = \lim_{h \rightarrow 0} \frac{(16+8h+h^2) - 16}{h} = \lim_{h \rightarrow 0} \frac{8h+h^2}{h} = \lim_{h \rightarrow 0} \frac{h(8+h)}{h} = \lim_{h \rightarrow 0} (8+h) = 8+0=8$$

$$21. \lim_{t \rightarrow 9} \frac{9-t}{3-\sqrt{t}} = \lim_{t \rightarrow 9} \frac{(3+\sqrt{t})(3-\sqrt{t})}{3-\sqrt{t}} = \lim_{t \rightarrow 9} (3+\sqrt{t}) = 3+\sqrt{9}=6$$

23.

$$\begin{aligned} \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7} &= \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7} \cdot \frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3} = \lim_{x \rightarrow 7} \frac{(x+2) - 9}{(x-7)(\sqrt{x+2} + 3)} \\ &= \lim_{x \rightarrow 7} \frac{x-7}{(x-7)(\sqrt{x+2} + 3)} = \lim_{x \rightarrow 7} \frac{1}{\sqrt{x+2} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6} \end{aligned}$$

29.

$$\begin{aligned} \lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) &= \lim_{t \rightarrow 0} \frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} = \lim_{t \rightarrow 0} \frac{(1 - \sqrt{1+t})(1 + \sqrt{1+t})}{t\sqrt{1+t}(1 + \sqrt{1+t})} = \lim_{t \rightarrow 0} \frac{-t}{t\sqrt{1+t}(1 + \sqrt{1+t})} \\ &= \lim_{t \rightarrow 0} \frac{-1}{\sqrt{1+t}(1 + \sqrt{1+t})} = \frac{-1}{\sqrt{1+0}(1 + \sqrt{1+0})} = -\frac{1}{2} \end{aligned}$$

35. $1 \leq f(x) \leq x^2 + 2x + 2$ for all x . Now $\lim_{x \rightarrow -1} 1 = 1$ and

$\lim_{x \rightarrow -1} (x^2 + 2x + 2) = \lim_{x \rightarrow -1} x^2 + 2 \lim_{x \rightarrow -1} x + \lim_{x \rightarrow -1} 2 = (-1)^2 + 2(-1) + 2 = 1$. Therefore, by the Squeeze Theorem,

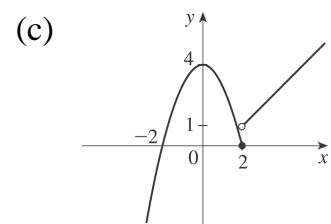
$$\lim_{x \rightarrow -1} f(x) = 1.$$

46. (a)

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} (4 - x^2) = \lim_{x \rightarrow 2^-} 4 - \lim_{x \rightarrow 2^-} x^2 \\ &= 4 - 4 = 0 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} (x - 1) = \lim_{x \rightarrow 2^+} x - \lim_{x \rightarrow 2^+} 1 \\ &= 2 - 1 = 1 \end{aligned}$$

(b) No, $\lim_{x \rightarrow 2} f(x)$ does not exist since $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$.



56. Let $f(x) = [x]$ and $g(x) = -[x]$. Then $\lim_{x \rightarrow 3} f(x)$ and $\lim_{x \rightarrow 3} g(x)$ do not exist (Example 10) but

$$\lim_{x \rightarrow 3} [f(x) + g(x)] = \lim_{x \rightarrow 3} ([x] - [x]) = \lim_{x \rightarrow 3} 0 = 0.$$