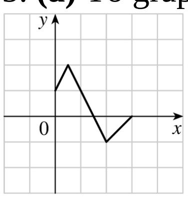
1. **(a)** If the graph of f is shifted 3 units upward, its equation becomes $y=f(x)+3$.
(b) If the graph of f is shifted 3 units downward, its equation becomes $y=f(x)-3$.
(c) If the graph of f is shifted 3 units to the right, its equation becomes $y=f(x-3)$.
(d) If the graph of f is shifted 3 units to the left, its equation becomes $y=f(x+3)$.
(e) If the graph of f is reflected about the x –axis, its equation becomes $y=-f(x)$.
(f) If the graph of f is reflected about the y –axis, its equation becomes $y=f(-x)$.
(g) If the graph of f is stretched vertically by a factor of 3 , its equation becomes $y=3f(x)$.
(h) If the graph of f is shrunk vertically by a factor of 3 , its equation becomes $y=\frac{1}{3}f(x)$.

3. **(a)** (graph 3) The graph of f is shifted 4 units to the right and has equation $y=f(x-4)$.
(b) (graph 1) The graph of f is shifted 3 units upward and has equation $y=f(x)+3$.
(c) (graph 4) The graph of f is shrunk vertically by a factor of 3 and has equation $y=\frac{1}{3}f(x)$.
(d) (graph 5) The graph of f is shifted 4 units to the left and reflected about the x –axis. Its equation is $y=-f(x+4)$.
(e) (graph 2) The graph of f is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y=2f(x+6)$.

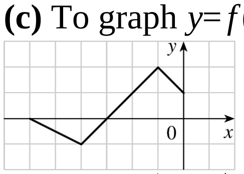
5. **(a)** To graph $y=f(2x)$ we shrink the graph of f horizontally by a factor of 2.



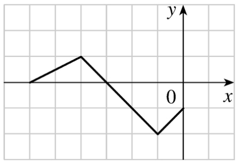
The point $(4,-1)$ on the graph of f corresponds to the point $\left(\frac{1}{2}\cdot 4,-1\right)=(2,-1)$.

(b) To graph $y=f\left(\frac{1}{2}x\right)$ we stretch the graph of f horizontally by a factor of 2.

The point $(4,-1)$ on the graph of f corresponds to the point $(2\cdot 4,-1)=(8,-1)$.
(c) To graph $y=f(-x)$ we reflect the graph of f about the y –axis.

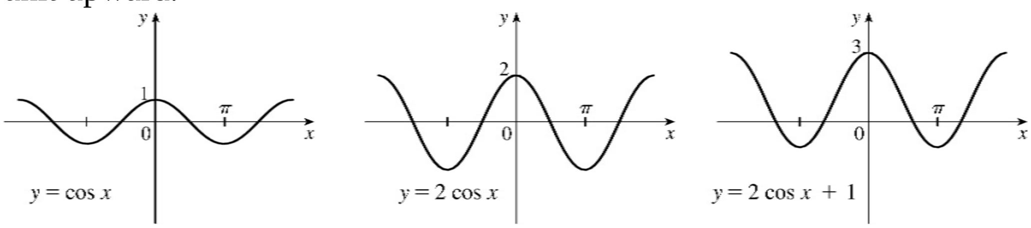


The point $(4,-1)$ on the graph of f corresponds to the point $(-1\cdot 4,-1)=(-4,-1)$.
(d) To graph $y=-f(-x)$ we reflect the graph of f about the y –axis, then about the x –axis.



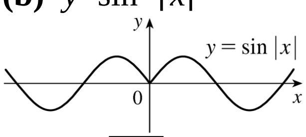
The point $(4,-1)$ on the graph of f corresponds to the point $(-1\cdot 4,-1\cdot -1)=(-4,1)$.

13. $y=1+2\cos\ x$: Start with the graph of $y=\cos\ x$, stretch vertically by a factor of 2, and then shift 1 unit upward.

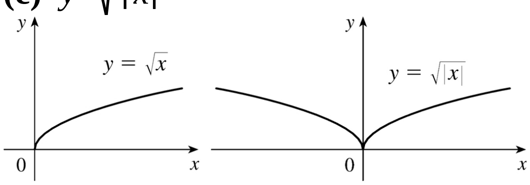


27. **(a)** To obtain $y=f(|x|)$, the portion of the graph of $y=f(x)$ to the right of the y –axis is reflected about the y –axis.

(b) $y=\sin\ |x|$



(c) $y=\sqrt{|x|}$



37. $f(x)=\sin\ x$, $D=\mathbb{R}$; $g(x)=1-\sqrt{x}$, $D=[0,\infty)$.

$(f\circ g)(x)=f(g(x))=f(1-\sqrt{x})=\sin\ \left(1-\sqrt{x}\right)$, $D=[0,\infty]$.

$(g\circ f)(x)=g(f(x))=g(\sin\ x)=1-\sqrt{\sin\ x}$. for $\sqrt{\sin\ x}$ to be defined, we must have $\sin\ x\geq 0\Leftrightarrow$ $x\in[0,\pi]\cup[2\pi,3\pi]\cup[-2\pi,-\pi]\cup[4\pi,5\pi]\cup[-4\pi,-3\pi]\cup\ \ldots$, so $D=\{x|x\in[2n\pi,\pi+2n\pi],\text{where}\ \ n\ \text{ is an integer}\}$.

$(f\circ f)(x)=f(f(x))=f(\sin\ x)=\sin\ (\sin\ x)$, $D=\mathbb{R}$.

$(g\circ g)(x)=g(g(x))=g\left(1-\sqrt{x}\right)=1-\sqrt{1-\sqrt{x}}$,

$D=\left\{x\geq 0|1-\sqrt{x}\geq 0\right\}=\left\{x\geq 0|1\geq\sqrt{x}\right\}=\left\{x\geq 0|\sqrt{x}\leq 1\right\}=[0,1]$.

39. $f(x)=x+\dfrac{1}{x}$, $D=\{x|x\neq 0\}$; $g(x)=\dfrac{x+1}{x+2}$, $D=\{x|x\neq -2\}$.

$$f\circ g(x)=f(g(x))=f\left(\frac{x+1}{x+2}\right)=\frac{x+1}{x+2}+\frac{1}{\frac{x+1}{x+2}}=\frac{x+1}{x+2}+\frac{x+2}{x+1}$$

$$=\frac{(x+1)(x+1)+(x+2)(x+2)}{(x+2)(x+1)}=\frac{\left(x^2+2x+1\right)+\left(x^2+4x+4\right)}{(x+2)(x+1)}=\frac{2x^2+6x+5}{(x+2)(x+1)}$$

Since $g(x)$ is not defined for $x=-2$ and $f(g(x))$ is not defined for $x=-2$ and $x=-1$, the domain of $(f\circ g)(x)$ is $D=\{x|x\neq -2,-1\}$.

$$(g\circ f)(x)=g(f(x))=g\left(x+\frac{1}{x}\right)=\frac{\left(x+\frac{1}{x}\right)+1}{\left(x+\frac{1}{x}\right)+2}=\frac{\frac{x^2+1+x}{x}}{\frac{x^2+1+2x}{x}}=\frac{x^2+x+1}{x^2+2x+1}=\frac{x^2+x+1}{(x+1)^2}\ .$$

Since $f(x)$ is not defined for $x=0$ and $g(f(x))$ is not defined for $x=-1$, the domain of $(g\circ f)(x)$ is $D=\{x|x\neq -1,0\}$.

$$(f\circ f)(x)=f(f(x))=f\left(x+\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)+\frac{1}{x+\frac{1}{x}}=x+\frac{1}{x}+\frac{1}{\frac{x^2+1}{x}}=x+\frac{1}{x}+\frac{x}{x^2+1}$$

$$=\frac{x(x)\left(x^2+1\right)+1\left(x^2+1\right)+x(x)}{x(x^2+1)}=\frac{x^4+x^2+x^2+1+x^2}{x(x^2+1)}$$

$$=\frac{x^4+3x^2+1}{x(x^2+1)}\ ,\ D=\{x|x\neq 0\}\ .$$

$$(g\circ g)(x)=g(g(x))=g\left(\frac{x+1}{x+2}\right)=\frac{\frac{x+1}{x+2}+1}{\frac{x+1}{x+2}+2}=\frac{\frac{x+1+1(x+2)}{x+2}}{\frac{x+1+2(x+2)}{x+2}}=\frac{x+1+x+2}{x+1+2x+4}=\frac{2x+3}{3x+5}$$. Since $g(x)$ is not defined for $x=-2$ and $g(g(x))$ is not defined for $x=-\dfrac{5}{3}$, the domain of $(g\circ g)(x)$ is $D=\left\{x|x\neq -2,-\dfrac{5}{3}\right\}$.

53. Let $h(x)=\sqrt{x}$, $g(x)=\sec x$, and $f(x)=x^4$. Then $(f\circ g\circ h)(x)=\left(\sec\sqrt{x}\right)^4=\sec^4\left(\sqrt{x}\right)=H(x)$.

54. **(a)** $f(g(1))=f(6)=5$
(b) $g(f(1))=g(3)=2$
(c) $f(f(1))=f(3)=4$
(d) $g(g(1))=g(6)=3$
(e) $(g\circ f)(3)=g(f(3))=g(4)=1$
(f) $(f\circ g)(6)=f(g(6))=f(3)=4$

55. **(a)** $g(2)=5$, because the point $(2,5)$ is on the graph of g. Thus, $f(g(2))=f(5)=4$, because the point $(5,4)$ is on the graph of f.
(b) $g(f(0))=g(0)=3$
(c) $(f\circ g)(0)=f(g(0))=f(3)=0$
(d) $(g\circ f)(6)=g(f(6))=g(6)$. This value is not defined, because there is no point on the graph of g that has x –coordinate 6.
(e) $(g\circ g)(-2)=g(g(-2))=g(1)=4$
(f) $(f\circ f)(4)=f(f(4))=f(2)=-2$

57. **(a)** Using the relationship *distance* = *rate* · *time* with the radius r as the distance, we have $r(t)=60t$.
(b) $A=\pi r^2 \Rightarrow (A\circ r)(t)=A(r(t))=\pi(60t)^2=3600\pi t^2$. This formula gives us the extent of the rippled area (in cm^2) at any time t.