

1. (a) The point  $(-1, -2)$  is on the graph of  $f$ , so  $f(-1) = -2$ .  
 (b) When  $x = 2$ ,  $y$  is about 2.8, so  $f(2) \approx 2.8$ .  
 (c)  $f(x) = 2$  is equivalent to  $y = 2$ . When  $y = 2$ , we have  $x = -3$  and  $x = 1$ .  
 (d) Reasonable estimates for  $x$  when  $y = 0$  are  $x = -2.5$  and  $x = 0.3$ .  
 (e) The domain of  $f$  consists of all  $x$ -values on the graph of  $f$ . For this function, the domain is  $-3 \leq x \leq 3$ , or  $[-3, 3]$ . The range of  $f$  consists of all  $y$ -values on the graph of  $f$ . For this function, the range is  $-2 \leq y \leq 3$ , or  $[-2, 3]$ .  
 (f) As  $x$  increases from  $-1$  to  $3$ ,  $y$  increases from  $-2$  to  $3$ . Thus,  $f$  is increasing on the interval  $[-1, 3]$ .

$$19. f(x) = 3x^2 - x + 2.$$

$$f(2) = 3(2)^2 - 2 + 2 = 12 - 2 + 2 = 12.$$

$$f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16.$$

$$f(a) = 3a^2 - a + 2.$$

$$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2.$$

$$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a - 1 + 2 = 3a^2 + 5a + 4.$$

$$2f(a) = 2 \cdot f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4.$$

$$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2.$$

$$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3(a^4) - a^2 + 2 = 3a^4 - a^2 + 2.$$

$$\begin{aligned} [f(a)]^2 &= [3a^2 - a + 2]^2 = (3a^2 - a + 2)(3a^2 - a + 2) \\ &= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 13a^2 - 4a + 4. \end{aligned}$$

$$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2.$$

$$21. f(x) = x - x^2, \text{ so } f(2+h) = 2+h - (2+h)^2 = 2+h - (4+4h+h^2) = 2+h-4-4h-h^2 = -(h^2+3h+2),$$

$$f(x+h) = x+h - (x+h)^2 = x+h - (x^2+2xh+h^2) = x+h-x^2-2xh-h^2, \text{ and}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{x+h-x^2-2xh-h^2-x+x^2}{h} = \frac{h-2xh-h^2}{h} = \frac{h(1-2x-h)}{h} = 1-2x-h.$$

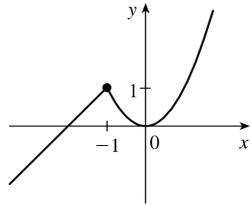
$$23. f(x) = x/(3x-1) \text{ is defined for all } x \text{ except when } 0 = 3x-1 \Leftrightarrow x = \frac{1}{3}, \text{ so the domain is}$$

$$\left\{ x \in \mathbb{R} \mid x \neq \frac{1}{3} \right\} = \left( -\infty, \frac{1}{3} \right) \cup \left( \frac{1}{3}, \infty \right).$$

27.  $h(x) = 1/\sqrt[4]{x^2 - 5x}$  is defined when  $x^2 - 5x > 0 \Leftrightarrow x(x-5) > 0$ . Note that  $x^2 - 5x \neq 0$  since that would result in division by zero. The expression  $x(x-5)$  is positive if  $x < 0$  or  $x > 5$ . (See Appendix A for methods for solving inequalities.) Thus, the domain is  $(-\infty, 0) \cup (5, \infty)$ .

$$39. f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Note that for  $x = -1$ , both  $x+2$  and  $x^2$  are equal to 1. Domain is  $\mathbb{R}$ .



45. For  $-1 \leq x \leq 2$ , the graph is the line with slope 1 and  $y$ -intercept 1, that is, the line  $y = x + 1$ . For  $2 < x \leq 4$ , the graph is the line with slope  $-\frac{3}{2}$  and  $x$ -intercept 4, so  $y - 0 = -\frac{3}{2}(x - 4) = -\frac{3}{2}x + 6$ . So the

$$\text{function is } f(x) = \begin{cases} x+1 & \text{if } -1 \leq x \leq 2 \\ -\frac{3}{2}x+6 & \text{if } 2 < x \leq 4 \end{cases}$$

49. Let the length of a side of the equilateral triangle be  $x$ . Then by the Pythagorean Theorem, the height  $y$  of the triangle satisfies  $y^2 + \left(\frac{1}{2}x\right)^2 = x^2$ , so that  $y^2 = x^2 - \frac{1}{4}x^2 = \frac{3}{4}x^2$  and  $y = \frac{\sqrt{3}}{2}x$ . Using the formula for the area  $A$  of a triangle,  $A = \frac{1}{2}(\text{base})(\text{height})$ , we obtain  $A(x) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$ , with domain  $x > 0$ .

53. The height of the box is  $x$  and the length and width are  $L = 20 - 2x$ ,  $W = 12 - 2x$ . Then  $V = LWx$  and so  $V(x) = (20 - 2x)(12 - 2x)(x) = 4(10 - x)(6 - x)(x) = 4x(60 - 16x + x^2) = 4x^3 - 64x^2 + 240x$ . The sides  $L$ ,  $W$ , and  $x$  must be positive. Thus,  $L > 0 \Leftrightarrow 20 - 2x > 0 \Leftrightarrow x < 10$ ;  $W > 0 \Leftrightarrow 12 - 2x > 0 \Leftrightarrow x < 6$ ; and  $x > 0$ . Combining these restrictions gives us the domain  $0 < x < 6$ .