2.5

#43 Given: $f(x) = x^3 - x^2 + x$

Show: There exists a number c such that $f(c) = 10$.

Proof: To show this, we will use the Intermediate Value Theorem (IVT) with $N = 10$. We will need to find numbers a & b such that $f(a) < N < f(b)$. If $a = 2$ then $f(a) = 2^3 - 2^2 + 2 = 8 - 4 + 2 = 6$ and if $b = 3$ then $f(b) = 3^3 - 3^2 + 3 = 21$. We see that $\underset{f(a)}{6} < \underset{N}{10} < \underset{f(b)}{21}$ when $a = 2$ and $b = 3$. Next, f is a 3rd degree polynomial and all polynomials are continuous everywhere. In particular, f must be continuous on $[2,3]$. We observe that $f(a) \neq f(b)$ since $6 \neq 21$. Thus all three conditions in the hypothesis of the IVT are satisfied. Because of this, the IVT guarantees us the existence of a number c in $(2,3)$ such that $f(c) = 10 = N$. This completes the proof.