1. Let $u=3x$. Then $du=3\,dx$, so $dx=\frac{1}{3}\,du$. Thus,
$\int \cos 3x\,dx=\int \cos u\left(\frac{1}{3}\,du\right)=\frac{1}{3}\int \cos u\,du=\frac{1}{3}\sin u+C=\frac{1}{3}\sin 3x+C$. Don't forget that it is often very easy to check an indefinite integration by differentiating your answer. In this case,
$\frac{d}{dx}\left(\frac{1}{3}\sin 3x+C\right)=\frac{1}{3}(\cos 3x)\cdot 3=\cos 3x$, the desired result.

3. Let $u=x^3+1$. Then $du=3x^2\,dx$ and $x^2\,dx=\frac{1}{3}\,du$, so
$\int x^2\sqrt{x^3+1}\,dx=\int \sqrt{u}\left(\frac{1}{3}\,du\right)=\frac{1}{3}\,\frac{u^{3/2}}{3/2}+C=\frac{1}{3}\cdot\frac{2}{3}\,u^{3/2}+C=\frac{2}{9}(x^3+1)^{3/2}+C.$

6. Let $u=\cos\theta$. Then $du=-\sin\theta\,d\theta$ and $\sin\theta\,d\theta=-du$, so
$\int \cos^4\theta\,\sin\theta\,d\theta=\int u^4(-du)=-\frac{1}{5}u^5+C=-\frac{1}{5}\cos^5\theta+C.$

11. Let $u=1+x+2x^2$. Then $du=(1+4x)\,dx$, so
$\int \frac{1+4x}{\sqrt{1+x+2x^2}}\,dx=\int \frac{du}{\sqrt{u}}=\int u^{-1/2}\,du=\frac{u^{1/2}}{1/2}+C=2\sqrt{1+x+2x^2}+C.$

15. Let $u=4-t$. Then $du=-dt$ and $dt=-du$, so $\int \sqrt{4-t}\,dt=\int u^{1/2}(-du)=-\frac{2}{3}u^{3/2}+C=-\frac{2}{3}(4-t)^{3/2}+C.$

19. Let $u=\sqrt{t}$. Then $du=\frac{dt}{2\sqrt{t}}$ and $\frac{1}{\sqrt{t}}\,dt=2\,du$, so $\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt=\int \cos u\,(2\,du)=2\sin u+C=2\sin\sqrt{t}+C.$

21. Let $u=\sin\theta$. Then $du=\cos\theta\,d\theta$, so $\int \cos\theta\,\sin^6\theta\,d\theta=\int u^6\,du=\frac{1}{7}u^7+C=\frac{1}{7}\sin^7\theta+C.$

27. Let $u=\sec x$. Then $du=\sec x\tan x\,dx$, so
$\int \sec^3 x\tan x\,dx=\int \sec^2 x(\sec x\tan x)\,dx=\int u^2\,du=\frac{1}{3}u^3+C=\frac{1}{3}\sec^3 x+C.$

28. Let $u=x^3+1$. Then $x^3=u-1$ and $du=3x^2\,dx$, so

$$\int \sqrt[3]{x^3+1}\,x^5\,dx = \int \sqrt[3]{x^3+1}\cdot x^3\cdot x^2\,dx=\int u^{1/3}(u-1)\left(\frac{1}{3}\,du\right)=\frac{1}{3}\int (u^{4/3}-u^{1/3})\,du$$

$$= \frac{1}{3}\left(\frac{3}{7}u^{7/3}-\frac{3}{4}u^{4/3}\right)+C=\frac{1}{7}(x^3+1)^{7/3}-\frac{1}{4}(x^3+1)^{4/3}+C$$

32. Let $u=1-x$. Then $x=1-u$ and $dx=-du$, so

$$\int \frac{x^2}{\sqrt{1-x}}\,dx = \int \frac{(1-u)^2}{\sqrt{u}}(-du)=-\int \frac{1-2u+u^2}{\sqrt{u}}\,du=-\int\left(u^{-1/2}-2u^{1/2}+u^{3/2}\right)du$$

$$= -\left(2u^{1/2}-2\cdot\frac{2}{3}u^{3/2}+\frac{2}{5}u^{5/2}\right)+C=-2\sqrt{1-x}+\frac{4}{3}(1-x)^{3/2}-\frac{2}{5}(1-x)^{5/2}+C$$

69. Let $u=\ln x$. Then $du=\dfrac{dx}{x}$, so $\displaystyle\int \frac{(\ln x)^2}{x}\,dx=\int u^2\,du=\frac{1}{3}u^3+C=\frac{1}{3}(\ln x)^3+C.$

71. Let $u=1+e^x$. Then $du=e^x\,dx$, so $\displaystyle\int e^x\sqrt{1+e^x}\,dx=\int\sqrt{u}\,du=\frac{2}{3}u^{3/2}+C=\frac{2}{3}(1+e^x)^{3/2}+C$.

Or: Let $u=\sqrt{1+e^x}$. Then $u^2=1+e^x$ and $2u\,du=e^x\,dx$, so
$\displaystyle\int e^x\sqrt{1+e^x}\,dx=\int u\cdot 2u\,du=\frac{2}{3}u^3+C=\frac{2}{3}(1+e^x)^{3/2}+C.$

72. Let $u=\cos t$. Then $du=-\sin t\,dt$ and $\sin t\,dt=-du$, so $\displaystyle\int e^{\cos t}\sin t\,dt=\int e^u(-du)=-e^u+C=-e^{\cos t}+C.$

73. Let $u=\ln x$. Then $du=\dfrac{dx}{x}$, so $\displaystyle\int \frac{dx}{x\ln x}=\int\frac{du}{u}=\ln|u|+C=\ln|\ln x|+C.$

74. Let $u=e^x+1$. Then $du=e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x+1}\,dx=\int\frac{du}{u}=\ln|u|+C=\ln\left(e^x+1\right)+C.$