If $C'(x) < \bar{C}(x)$ then $\bar{C}(x)$ is decreasing. ~~Why?~~

Proof: Assume $C'(x) < \bar{C}(x)$

$\Rightarrow C'(x) < \frac{C(x)}{x}$ Since $\bar{C}(x) = \frac{C(x)}{x}$

$\Rightarrow x \cdot C'(x) < C(x)$ mult. both sides by x

$\Rightarrow x \cdot C'(x) - C(x) < 0$ subtract $C(x)$ on both sides

(★) $\Rightarrow \frac{x \cdot C'(x) - C(x)}{x^2} < \frac{0}{x^2} = 0$ divide both sides by x^2.

This is the marginal average cost function.

$\Rightarrow \bar{C}'(x) < 0$

$\Rightarrow \bar{C}(x)$ has negative slope

$\Rightarrow \bar{C}(x)$ must be going down

$\Rightarrow \bar{C}(x)$ is decreasing

(★) Note: If $\bar{C}(x) = \frac{C(x)}{x}$ then $\bar{C}'(x) = \frac{x \cdot C'(x) - C(x)(1)}{x^2}$

This is the quotient rule.