## Round 1

1.  Let P be the point of intersection of the lines with equations 2x + 3y = 1 and 3x - 2y = 21. Write an equation of the line passing through P and the origin.

    ANSWER $y = -\frac{3}{5}x$

2.  The supplement of a nonzero angle is k times the angle's complement. If k is a positive integer, what is its smallest possible value?

    ANSWER     3

3.  State the ones digit of the standard form of the number $2^{1234567}$ ?

    ANSWER     8

**Round 2**

1.  A triangle has sides with lengths $x$, $2x$, and $x\sqrt{7}$. Find the exact degree measure of the largest angle of the triangle, and the measures of the other angles to the nearest tenth of a degree.

ANSWER   120, 40.9, 19.1

2.  At a certain school there are 3 times as many boys as girls and 9 times as many girls as teachers. Let b represent the numbers of boys at the school. Express the total number of boys, girls and teachers in terms of b.

ANSWER   $\frac{37}{27}b$

3.  The points (-3,1), (2,5) and (1,-4) are three vertices of a square in the xy-plane. State the coordinates of the fourth vertex.

ANSWER   (6,0)

**Round 3**

1.  Let $c$ be a constant.  For what values of $c$ will the simultaneous equations $\begin{aligned} x - y &= 2 \\ cx + y &= 3 \end{aligned}$ have a solution in

    Quadant I?

    ANSWER     $-1 < c < \frac{3}{2}$

2.  State the measure, in radians, of the acute angle formed by the hands of an analog clock when the time is exactly 5:15pm.

    ANSWER     $\frac{3\pi}{8} rad$

3.  For the following diagram, trapezoid ABCD has $\overline{AB} \parallel \overline{DC}$, and median $\overline{EF}$, $\overline{AB}$ measures x units and $\overline{DC}$ measures 2x units.  Find the ratio of the area of trapezoid ABFE to the area of trapezoid ABCD.
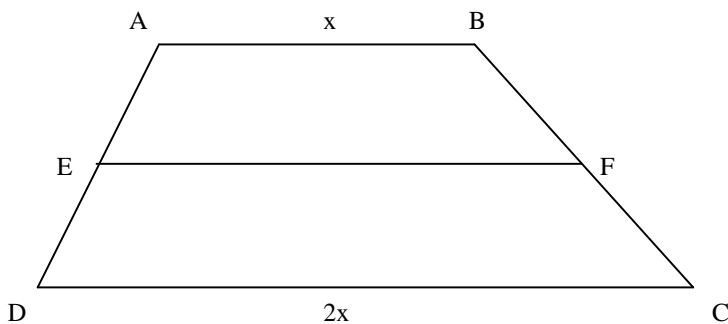


    ANSWER     $\frac{5}{12}$

**Round 4**

1. Determine the probability that, of three people selected at random, at least two were born on the same day.

ANSWER $\frac{19}{49}$

2. A puppy and two kittens weigh a total of 12 pounds. The puppy and larger kitten together weigh exactly twice the smaller kitten. The puppy and smaller kitten together weigh exactly the same as the larger kitten. What is the puppy's weight?

ANSWER 2 pounds

3. Suppose that $[A]^n$ represents raising the matrix $A$ to the $n^{th}$ power and $[A]^{\bullet n}$ represents raising each element of the matrix to the $n^{th}$ power.

Solve for x: $\det\left[\begin{pmatrix} 3 & 2 \\ x & 4 \end{pmatrix}^2\right] = \det\left[\begin{pmatrix} 3 & 2 \\ x & 4 \end{pmatrix}^{\bullet 2}\right]$

ANSWER x=0,x=6

## Round 5

1. During a period of days, it was observed that when it rained in the afternoon, it had been clear in the morning, and when it rained in the morning it was clear in the afternoon. It rained on 9 days and was clear on 6 afternoons and 7 mornings. How long was the period?

ANSWER     11 days

2. What is the sum of the digits of the standard form of the following product: $\left(2^{99}\right)\left(5^{103}\right)$

ANSWER     13

3. Find the measure (to the nearest tenth of a degree) of the acute angle of intersection between the lines $\frac{x}{4} - \frac{y}{7} = 1$ and $5x = -3y + 2$.

ANSWER     60.7

## Round 6

1. Consider the infinite sequence of equilateral triangles with side lengths $a, \frac{a}{2}, \frac{a}{4}, \frac{a}{8}, \dots$ Find the sum of the areas of the triangles.

ANSWER $\frac{a^2\sqrt{3}}{3}$

2. The front wheels of a wagon measure 3.5 feet in diameter. The rear wheels measure 4.25 feet in diameter. When the wagon is stopped, a chalk mark is made on a front wheel and a back wheel. Determine, exactly, how far the wagon will travel before both chalk marks return to their original positions at the same time.

ANSWER $\frac{119}{2}\pi$ feet

3. A rocket is fired vertically from ground level. A camera that will observe the rocket's flight is positioned at ground level 3000 feet from the launch pad. If the rocket is rising at the constant rate of 100 feet per second, express the camera's angle of elevation as a function of $t$, the time (in seconds) since liftoff.

ANSWER $\Theta = \tan^{-1}\left(\frac{t}{30}\right)$

**Round 7**

1. ABCD is a square and M and N are the midpoints of BC and CD respectively. Find the value of $\sin(\theta)$.
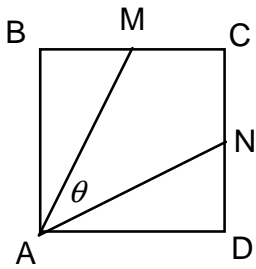


ANSWER   $\frac{3}{5}$

2. Two circles with radii $r_1$ and $r_2$ are centered at the origin in the xy-plane. One of the circles passes through the point $(1, m)$, and the line $y = mx + 4$ is tangent to the other circle. Find the value of the product $r_1 r_2$.

ANSWER   4

3. A large pump can fill a 22500-gallon tank 5 hours faster than a small pump. The large pump outputs water 150 gallons per hour faster than the small pump. Find the number of gallons pumped in 1 hour by the small pump.

ANSWER   750