Round 1

1. Let P be the point of intersection of the lines with equations 2x + 3y = 1 and 3x - 2y = 21. Write an equation of the line passing through P and the origin.

ANSWER _____

2. The supplement of a nonzero angle is k times the angle's complement. If k is a positive integer, what is its smallest possible value?

ANSWER _____

3. State the ones digit of the standard form of the number $2^{1234567}$?

Round 2

1. A triangle has sides with lengths x, 2x, and $x\sqrt{7}$. Find the exact degree measure of the largest angle of the triangle, and the measures of the other angles to the nearest tenth of a degree.

ANSWER _____

2. At a certain school there are 3 times as many boys as girls and 9 times as many girls as teachers. Let b represent the numbers of boys at the school. Express the total number of boys, girls and teachers in terms of b.

ANSWER _____

3. The points (-3,1), (2,5) and (1,-4) are three vertices of a square in the xy-plane. State the coordinates of the fourth vertex.

Round 3

1. Let *c* be a constant. For what values of *c* will the simultaneous equations $\begin{cases} x-y=2\\ cx+y=3 \end{cases}$ have a solution in

Quadant I?

ANSWER _____

2. State the measure, in radians, of the acute angle formed by the hands of an analog clock when the time is exactly 5:15pm.

ANSWER _____

3. For the following diagram, trapezoid ABCD has $\overline{AB} \parallel \overline{DC}$, and median \overline{EF} , \overline{AB} measures x units and \overline{DC} measures 2x units. Find the ratio of the area of trapezoid ABFE to the area of trapezoid ABCD.



Round 4

1. Determine the probability that, of three people selected at random, at least two were born on the same day.

ANSWER _____

2. A puppy and two kittens weigh a total of 12 pounds. The puppy and larger kitten together weigh exactly twice the smaller kitten. The puppy and smaller kitten together weigh exactly the same as the larger kitten. What is the puppy's weight?

ANSWER _____

3. Suppose that $[A]^n$ represents raising the matrix A to the nth power and $[A]^{\bullet n}$ represents raising each element of the matrix to the nth power.

Solve for x: $\det \begin{bmatrix} \begin{pmatrix} 3 & 2 \\ x & 4 \end{bmatrix}^2 = \det \begin{bmatrix} \begin{pmatrix} 3 & 2 \\ x & 4 \end{bmatrix}^{\bullet^2}$

Round 5

1. During a period of days, it was observed that when it rained in the afternoon, it had been clear in the morning, and when it rained in the morning it was clear in the afternoon. It rained on 9 days and was clear on 6 afternoons and 7 mornings. How long was the period?

ANSWER _____

2. What is the sum of the digits of the standard form of the following product: $(2^{99})(5^{103})$

ANSWER _____

3. Find the measure (to the nearest tenth of a degree) of the acute angle of intersection between the lines $\frac{x}{4} - \frac{y}{7} = 1$ and 5x = -3y + 2.

Round 6

1. Consider the infinite sequence of equilateral triangles with side lengths $a, \frac{a}{2}, \frac{a}{4}, \frac{a}{8}, \dots$ Find the sum of the areas of the triangles.

ANSWER _____

2. The front wheels of a wagon measure 3.5 feet in diameter. The rear wheels measure 4.25 feet in diameter. When the wagon is stopped, a chalk mark is made on a front wheel and a back wheel. Determine, exactly, how far the wagon will travel before both chalk marks return to their original positions at the same time.

ANSWER _____feet

3. A rocket is fired vertically from ground level. A camera that will observe the rocket's flight is positioned at ground level 3000 feet from the launch pad. If the rocket is rising at the constant rate of 100 feet per second, express the camera's angle of elevation as a function of t, the time (in seconds) since liftoff.

Round 7

1. ABCD is a square and M and N are the midpoints of BC and CD respectively. Find the value of $sin(\theta)$.



ANSWER _____

2. Two circles with radii r_1 and r_2 are centered at the origin in the xy-plane. One of the circles passes through the point (1,m), and the line y = mx + 4 is tangent to the other circle. Find the value of the product r_1r_2 .

ANSWER _____

3. A large pump can fill a 22500-gallon tank 5 hours faster than a small pump. The large pump outputs water 150 gallons per hour faster than the small pump. Find the number of gallons pumped in 1 hour by the small pump.

Group Round – The Revenge of π

Using only π 's, the four arithmetic operations, +, -, ×, /, the ceiling function (perhaps repeatedly), and parentheses, it is possible to write the integer 41 as follows:

$$ceiling\left(\pi \cdot ceiling\left(\frac{\pi \cdot \pi \cdot \pi + \pi}{\pi}\right) + \pi + \pi\right) = 41$$

Recall that $\operatorname{ceiling}(x)$ is the least integer greater than or equal to x.

For example: $\operatorname{ceiling}(5) = 5$ $\operatorname{ceiling}(5.3) = 6$ $\operatorname{ceiling}(-5.3) = -5$

But, there may be a way to write 41 using a smaller number of π 's.

GOAL:

Using only π 's, the four arithmetic operations, +, -, ×, /, the ceiling function (perhaps repeatedly), and parentheses, write expressions representing the integers from 1 to 20 inclusive using as few π 's as possible.

Example: $ceiling(\pi) \cdot ceiling(\pi + \pi) + ceiling(\pi \cdot ceiling(\pi))$ would also be acceptable for 41. However, it is a "better" expression because it uses five π 's rather than the eight in the first example.

Non-Example: $ceiling(\pi \cdot \pi \cdot \pi + 9)$ is also equivalent to 41. However, it is not an acceptable expression for 41 because it uses a number other than π .

Non-Example: $ceiling(ceiling(\pi^{\pi})+\pi)$ is also equivalent to 41. However, it is not an acceptable expression for 41 because it uses an exponent.

SCORING:

ONE (1) point will be awarded **for every two correct responses** for the integers 1-20 inclusive. Multiple correct responses for any single integer will not receive extra credit.

THREE (3) bonus points will be awarded to the first team with correct responses for each integer 1-20 inclusive. TWO (2) bonus points will be awarded to the second team with correct responses for each integer 1-20 inclusive. ONE (1) bonus point will be awarded to the third team with correct responses for each integer 1-20 inclusive.

FIVE (5) bonus points will be awarded to the team with correct responses for each integer 1-20 inclusive and uses the least total number of π 's. THREE (3) bonus points will be awarded to the team with correct responses for each integer 1-20 inclusive and uses the second least total number of π 's. ONE (1) bonus point will be awarded to the team with correct responses for each integer 1-20 inclusive and uses the third least total number of π 's. In case of a tie, the team that submits their answers earlier will receive the larger number of bonus points.

You are to turn in one answer sheet per team. You may abbreviate ceiling(x) as C(x).

1=	11=
2=	12=
3=	13=
4=	14=
5=	15=
6=	16=
7=	17=
8=	18=
9=	19=
10=	20=

School _____