Practice Round

1. A small leak will fill an empty boat in 7 hours. The boat's bilge pump will empty a full boat in 4 hours. If the boat is half full and leaking, how long would it take the pump to empty the boat?



2. What is the ones digit of 47^{2006} ?

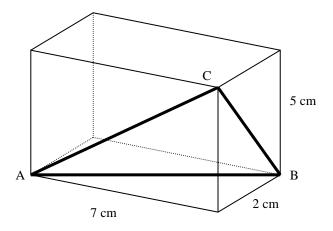
Answer: 9

Answer: $\frac{14}{3}$ hours

3. Consider the square defined by the boundaries $0 \le X \le 1$ and $0 \le Y \le 1$. If a point (X, Y) is randomly chosen from the square, find the probability that $Y < \frac{1}{2}$, given Y > X.

Round 1

1. A rectangular solid has edges measuring 7 cm, 5 cm and 2 cm (see the figure). Find the measure (to the nearest tenth of a degree) of $\angle ABC$. Note: Figure not drawn to scale.



Answer: 84.1 degrees

2. An equilateral triangle and a regular hexagon have the same perimeter. What is the ratio of the area of the hexagon to the area of the triangle?

Answer: $\frac{3}{2}$

3. Suppose that for a company the mean salary is \$42000, the median salary is \$40000, and the standard deviation of the salaries is \$10000. Suppose that everyone in the company receives a pay increase of \$3000. What will be the new mean, median and standard deviation of the company salaries?

Round 2

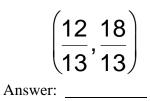
1. The sixth term of a geometric series is 6, and the 2000th term is 2000. Find the result when the 2006th term is divided by the 2005th term.

 $\left(\frac{1000}{3}\right)^{\frac{1}{1994}}$ Answer:

2. How many natural number divisors does 1200 have?

Answer: <u>30</u>

3. Find the point on the line 2x + 3y = 6 that is closest to the origin.



Round 3

1. A quadratic function f(x) has f(1) = 2, f(2) = 3. The minimum value of f(x) is 2. Find f(-2).

Answer: <u>11</u>

2. Write $\cos(\sin^{-1} x)$ as an algebraic function of x.

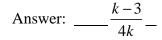
 $\sqrt{1-x^2}$ Answer:

3. The long side of a rectangular sheet of paper is less than twice as long as the short side. When it is folded in half along a line parallel to the short side the folded rectangle is geometrically similar to the original sheet. What is the ratio of the length of the long side to the length of the short side for the original sheet of paper?

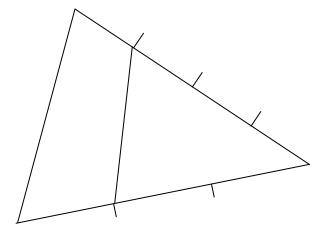
Answer: $\underline{\sqrt{2}}$

Round 4

1. Let *k* be an odd, natural number greater than 2. Consider the set $A = \{1, 2, 3, ..., k - 2, k - 1, k\}$. Find the probability that two numbers selected at random from *A* without replacement are even.



2. The sides of the large triangle have been divided into equal segments, as shown in the diagram. If the area of the large triangle is 12 square inches, find the area of the small triangle.



Answer: <u>6</u> square inches

3. You are late going to school from your home, so you take the drive at an average of 72 MPH to get there. After school is over, you are in no hurry to get home, so you drive the same route home at an average of 48 MPH. What is your average speed for the entire round trip?

Round 5

1. A regular n-gon is a polygon having n equal sides and n equal angles. Write a formula for the area of a regular n-gon that is inscribed in a circle of radius r. Your formula should be in terms of n and r.

Answer: $A = nr^2 sin(\pi/n) cos(\pi/n)$ (alternate forms: $0.5nr^2 sin(2\pi/n)$ and/or replace π by 180°)

2. A sequence has general term $sin\left(\frac{n\pi}{2}\right)$. Find the sum of the sequence as n goes from 1 to 999,997.

Answer: 1

3. Suppose that you are told that 16% of adult American women have cholesterol levels below 164 and that 2.5% have cholesterol levels above 236. Assuming that the normal distribution is being used to model the distribution, what would be the standard deviation. Round your answer to the nearest whole number.

Round 6

1. A right circular cylinder is inscribed in a sphere of radius R. Write the volume of the cylinder in terms of R and h, where h is the height of the cylinder.

Answer:
$$V = \pi R^2 h - \frac{1}{4} \pi h^3$$

2. Suppose that p is a positive number such that 0 . What is the 100th positive solution to the equation <math>sin(x) = p? Give your answer in terms of p.

Answer: $99\pi - \sin^{-1}(p)$

3. The function f(x) satisfies f(2+x) = f(2-x) for all real numbers x. If the equation f(x) = 0 has four distinct real roots, what is the sum of the roots?