Louis M. Edwards Mathematics Super Bowl
Valencia Community College -- April 13, 2007

Practice Round

1. An object placed in a room of constant temperature T_s warms or cools to approach the temperature of the room. The object's temperature $T(t)$ at time t is given by Newton's Law of Cooling, which says that $T(t) = T_s + Da^t$.

   Suppose that for a certain object in this situation, D<0. Is the object warming, cooling or at a steady state?

2. If $\frac{b}{a} = 2$ and $\frac{c}{b} = 3$ what is the ratio of a + b to b + c.

3. Suppose you play a game with the following conditions:
   - If you flip a coin and it lands face-up, you take one step to the left.
   - If you flip a coin and it lands face-down, you take one step to the right.
   - If, after 4 coin flips, you are in your original position; you win.

   Find the probability of winning the game.

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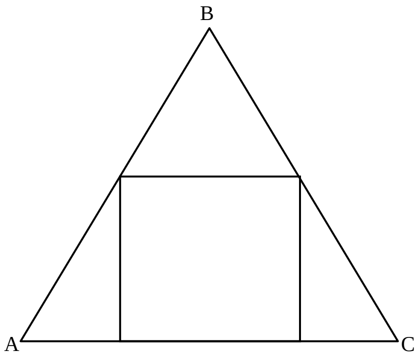
Round 1

1. Given the equilateral triangle ABC, with $m\overline{AB}=1$; find the area of the inscribed square.



2. The asymptotes of the hyperbola $\frac{x^2}{4}-\frac{y^2}{25}=1$ are the diagonals of a rectangle centered at the origin. An ellipse with major and minor axes parallel to the coordinate axes is inscribed in this rectangle. Find an equation of the ellipse.

3. Find the exact area of the triangle in the xy-plane with vertices $(0, 0)$, $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$, and $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$. Hint: The two points with nonzero coordinates lie on a circle centered at the origin.

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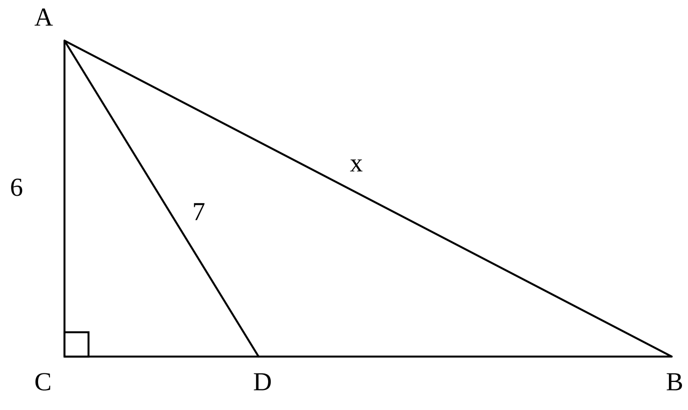
Round 2

1. Two friends play a game in which they take turns rolling a balanced six-sided die until a "3" occurs, and the winner is the person who rolls the "3". What is the probability that the person who has the first turn will win?

2. If f(x) = ax + b and $f^{-1}(x)$ = bx + a with a and b being real numbers, what is the value of a + b ?

3. Find the exact value of x in the figure, given that $m\angle CAD = m\angle DAB$, $m\overline{AC} = 6$, and $m\overline{AD} = 7$.



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Round 3

1. A parabola $y = ax^2 + bx + c$ has a vertex (4,2). If the point (2,0) is on the parabola, find the value of abc.

2. A circular cone has height 10 and radius 3. A right circular cylinder is sitting inside the cone. The base of the cylinder lies in the base of the cone, and the top circular edge of the cylinder just touches the surface of the cone. The cylinder has radius r and height h. Write the volume of the cylinder as a function of the single variable h.

3. If $A = \begin{bmatrix} x & 5 \\ x & -4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ y & y \end{bmatrix}$, and $AB = \begin{bmatrix} -6 & 1 \\ -33 & -26 \end{bmatrix}$; find the value of $x + y$.

Round 4

1. Suppose that 80% of teens own an iPod. If three teens are randomly selected what is the probability that less than half will have an iPod?

2. Suppose $f(x)$ is a one to one function, and $g(x) = f(x+2)$. If $f(0)=1$ and $f(1)=2$, find the value of $g^{-1}(1)+g^{-1}(2)$.

3. Consider the family of functions $f(x) = x^2 + kx$, where k is real number. The vertices of these parabolas can themselves be modeled by a quadratic function. Write the quadratic function that models the vertices of the family of functions.

Round 5

1. Let m be the slope of a line which passes through the point (2,1) and has positive x and y intercepts. Express the area of the triangle formed in the first quadrant by the line and the coordinate axes as a function of m.

2. Find the smallest whole number k such that $10k$ is a perfect square and $6k$ is a perfect cube.

3. Let P be the point (-2, 3) in the xy-plane. Find the exact radian measure of the smallest positive angle that is formed by the line segment from the origin to P and the positive x-axis.

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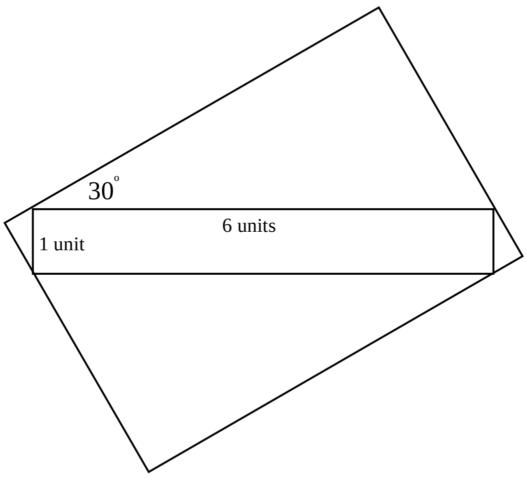
Round 6

1. A lot is in the shape of a right triangle with the following properties:
   - The longer leg is 7 meters shorter than the hypotenuse
   - The sum of the sides of the lot totals 392 meters.

   Find the measure of the shortest side of the lot.

2. Find the exact area of the circumscribed rectangle in the figure given below:



3. A string of holiday lights contains 25 bulbs. The bulbs are wired in series, so that if any bulb fails the whole string will go dark. Each light has a 2% probability of failing during a three year period. What is the probability (to the nearest tenth of a percent) the string of lights will remain bright for three years.

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Group Round

As a group, solve as many of the following problems as possible in the allotted time. Each problem is worth one (1) point.

The first team that submits at least eight correct responses will receive three (3) additional points.
The second team that submits at least eight correct responses will receive two (2) additional points.
The third team that submits at least eight correct responses will receive one (1) additional point.

1. A circle and a parabola intersect at the following four points: $(-4,15)$, $(-3,8)$, $(3,8)$, and $(4,15)$. Determine the equations of the circle and the parabola.

2. Dennis is driving his car (that only moves at one speed) across a bridge that stretches from point A to point B. At the moment he is $\frac{3}{8}$ of the way across the bridge he sees a truck barreling towards him (from the direction of point A) at 80 miles per hour. If he were to drive towards point A, the car and truck would meet at point A. If he were to drive to point B, the car and truck would meet at point B. How fast is Dennis' car?

3. Determine the integers *x* and *y* such that:
$$x+y+\sqrt{x+y}=56$$
$$x-y+\sqrt{x-y}=30$$

4. The integers *b* and *c* are chosen so that:
   The quadratic equation $5x^2+bx+c=0$ has a root of $x=2$
   The quadratic equation $5x^2+cx+b=0$ has a root of $x=3$
   Find the value of *b* and the value of *c*.

5. A *lattice point* is a point (x,y), where both of the coordinates are integers. For example $(1,2)$ and $(-7,0)$ are lattice points, but $(6,-1.05)$ is not. Determine the number of lattice points on the circumference of the circle defined by $x^2+y^2=100$.

6. Write a formula in terms of n for the sum $1\cdot 1!+2\cdot 2!+3\cdot 3!+\ldots n\cdot n!$

7. Find the number of integers *n* such that $\frac{3n^2-7n+138}{n-3}$ is also an integer.

8. A 6x6x6 cube has its faces painted blue. It is then cut into 216 1x1x1 cubes (some of which have blue faces). From the small cubes, a single cube is randomly chosen and rolled like a die. Find the probability that the small cube lands with a blue face up.

9. If $x^4 = A(x-1)(x-2)(x-3)(x-4)+B(x-1)(x-2)(x-3)+C(x-1)(x-2)+D(x-1)+E$, find the value of $A+B+C+D+E$

10. For the diagram below, the radius of the circle is 1. Each circular arc also has a radius equal to 1. Find the total area of the shaded regions bounded by the circular arcs.