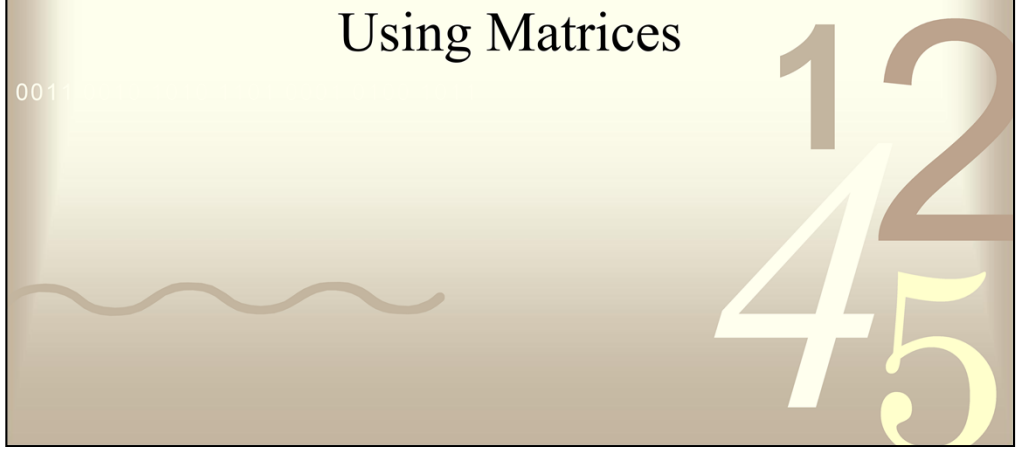
9.4 Solutions to Linear Systems Using Matrices



Matrices

- A matrix is a rectangular array of elements:

$$\begin{bmatrix} 1 & -2 & 4 \\ 5 & 1 & 3 \\ -1 & 4 & 7 \end{bmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- A matrix that has dimensions $m \times n$ has m rows and n columns.
- A square matrix has the same number of rows and columns ($n \times n$).

Example

Represent the system by an augmented matrix and state the dimensions of the matrix:

1. $\begin{cases} 3x - 5y = 4 \\ x + y = 2 \end{cases}$

2. $\begin{cases} 2x + y + 3z = 4 \\ -3x - y - 4z = 5 \\ x + y + 2z = 0 \end{cases}$

Row-Echelon Form

- A matrix is in row-echelon form if it has ones down the main diagonal and zeros beneath:

$$\begin{bmatrix} 1 & 2 & -5 & 7 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

- A matrix is in reduced row-echelon form if it has ones down the main diagonal and zeros above and below:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Examples

Write the equations that correspond to the augmented matrix, then solve the system:

1. $\begin{bmatrix} 1 & 4 & -2 \\ 0 & 1 & 3 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & -4 & 3/4 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -3 \end{bmatrix}$

Gaussian Elimination

MATRIX ROW TRANSFORMATIONS

For any augmented matrix representing a system of linear equations, the following row transformations result in an equivalent system of linear equations.

1. Any two rows may be interchanged.
2. The elements of any row may be multiplied by a nonzero constant.
3. Any row may be changed by adding to (or subtracting from) its elements a multiple of the corresponding elements of another row.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.783

Examples

Solve the system:

1. $\begin{cases} x+3y-2z=3 \\ -x-2y+z=-2 \\ 2x-7y+z=1 \end{cases}$

2. $\begin{cases} 4x-y-z=0 \\ 4x-2y=0 \\ 2x+z=1 \end{cases}$

3. $\begin{cases} 2x-y-z=0 \\ x-y-z=-2 \\ 3x-2y-2z=-2 \end{cases}$

4. $\begin{cases} 2x-4y-z=2 \\ x+y-3z=10 \\ -x-7y+8z=2 \end{cases}$

Example

Use technology to find the solution.
Approximate values to the nearest thousandth.

$$\begin{cases} 12x - 4y - 7z = 8 \\ -8x - 6y + 9z = 7 \\ 34x + 6y - 2z = 5 \end{cases}$$