

5.3 Exponential Functions & Models

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Form

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An exponential function is a function of the form

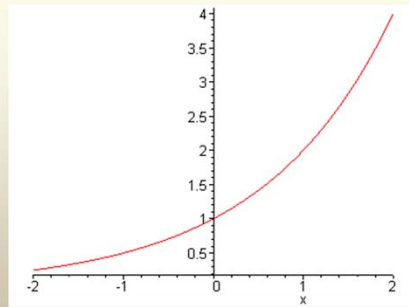
$$f(x) = b^x$$

where $b > 0$ and $b \neq 1$.

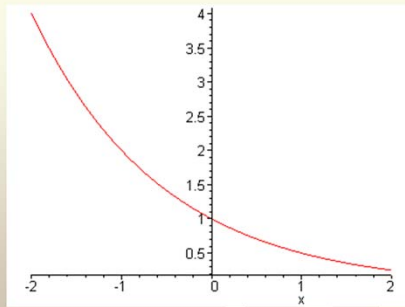


Shape of $y = b^x$

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$b > 1$
increasing



$0 < b < 1$
decreasing

Properties of the graph

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- Domain is $(-\infty, \infty)$
- Range is $(0, \infty)$
- Horizontal Asymptote is $y = 0$
- y -intercept is $(0, 1)$



Example

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- Graph $y = 2^x$
- Graph $y = 2^{-x}$
- Graph $y = 2^{x+1}$
- Graph $y = 2^{x+1}$



Compound Interest

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The amount accumulated in an account bearing interest compounded n times annually is

$$A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$

where P = principal invested

r = interest rate (as a decimal)

t = time in years



Example

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Suppose \$5000 is invested at an interest rate of 8%. Find the amount in the account after ten years if the interest is compounded

- a. annually
- b. semiannually
- c. daily



Definition of e

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e is an irrational number (like π)

$$e \approx 2.71828182845$$

The natural exponential function is

$$f(x) = e^x$$



Continuously Compounded Interest

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The amount accumulated in an account bearing interest compounded continuously is

$$A(t) = Pe^{rt}$$

where P = principal invested

r = interest rate (as a decimal)

t = time in years



Example

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\$5000 is invested at an interest rate of 8%, compounded continuously. How much is in the account after 10 years?

12
45

Exponential Growth and Decay

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Exponential growth of a population is given
by the formula

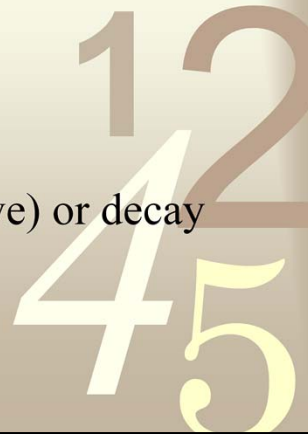
$$P(t) = P_0 e^{kt}$$

where

P_0 = initial size of the population

k = relative rate of growth (positive) or decay
(negative)

t = time



Example

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The population of Phoenix, Arizona, in 2000 was 1.3 million and growing continuously at a 3% rate.

- a. Assuming this trend continues, estimate the population of Phoenix in 2010.
- b. Determine graphically or numerically when this population might reach 2 million.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.416, problem 98.