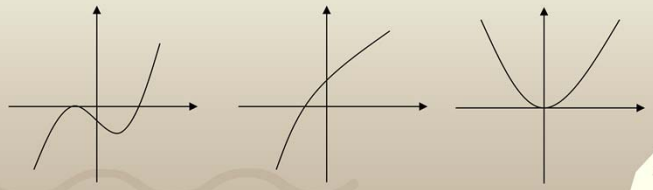
5.2 Inverse Functions & Their Representations

Horizontal Line Test

If no horizontal line intersects the graph of a function more than once, then the function is a one-to-one function.



Inverse of a function

If *f(x)* is a function, its inverse, denoted $f^{-1}(x)$, "undoes" what f "does".

That is, if $f(a) = b$ then $f^{-1}(b) = a$

Note: $f^{-1} \neq \frac{1}{f}$

Relationship between a function and its inverse

- The domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}.
- If (a, b) is an ordered pair on the graph of f, then (b, a) is an ordered pair on the graph of f^{-1}.
- The graph of f^{-1} is the graph of f reflected about the line $y = x$.

Relationship between a function and its inverse

- $f^{-1}(f(x)) = x$
- $f(f^{-1}(x)) = x$

To find the inverse of a function:

- Write the function as $y = f(x)$
- Exchange x & y
- Solve for y
- Write the solution using inverse notation

Example $f(x) = 5x$

$$y = 5x$$

$$x = 5y$$

$$y = \frac{1}{5}x$$

$$f^{-1}(x) = \frac{1}{5}x$$

Examples

For the function given, find $f^{-1}(x)$

a. $f(x) = 1 - \frac{1}{2}x^3$

b. $f(x) = \frac{x-1}{2}$

c. $f(x) = \frac{3x}{x-1}$

Examples

- Restrict the domain of $f(x) = x^4 - 1$ so that f is one-to-one. Then find its inverse.

- p. 396, problem 106