5.1 Combining Functions

Operations on Functions

For two functions, *f* and *g*, we define the new functions

$$(f + g)(x) = f(x) + g(x) \text{ with domain D}$$

$$(f - g)(x) = f(x) - g(x) \text{ with domain D}$$

$$(fg)(x) = f(x) \cdot g(x) \text{ with domain D}$$

$$(f/g)(x) = f(x) / g(x) \text{ with domain D and}$$

$$g(x) \neq 0$$

Where D is the domain that *f* and g have in common.

Examples

For the functions given, find $f+g, f-g, fg,$ and f/g.

A. $f(x) = \dfrac{1}{x}, \quad g(x) = x^3$
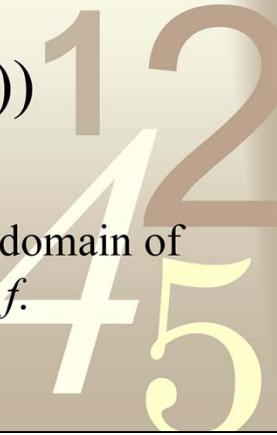
B. $f(x) = 6 - x, \quad g(x) = \sqrt{x-4}$

Composition of Functions

For the functions f and g, we define the composition function

$$(f \circ g)(x) = f(g(x))$$

The domain is all values of x in the domain of g for which $g(x)$ is in the domain of f.

Example

If $f(x) = \sqrt{4 - x}$ and $g(x) = x^2$, find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

State the domain of each

Example

If $f(x) = \dfrac{1}{3x}$ and $g(x) = \dfrac{2}{x-1}$, find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

State the domain of each