4.8 Power Functions & Radical Equations

PROPERTIES OF RATIONAL EXPONENTS

001 Let m and n be positive integers with $\frac{m}{n}$ in lowest terms and $n \geq 2$. Let r and p be rational numbers. Assume that b is a nonzero real number and that each expression is a real number.

Property	***Example***
1. $b^{m/n} = (b^m)^{1/n} = (b^{1/n})^m$	$4^{3/2} = (4^3)^{1/2} = (4^{1/2})^3 = 2^3 = 8$
2. $b^{m/n} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$	$8^{2/3} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$
3. $(b^r)^p = b^{rp}$	$(2^{3/2})^4 = 2^6 = 64$
4. $b^{-r} = \frac{1}{b^r}$	$4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{2}$
5. $b^r b^p = b^{r+p}$	$3^{5/2} \cdot 3^{3/2} = 3^{(5/2)+(3/2)} = 3^4 = 81$
6. $\frac{b^r}{b^p} = b^{r-p}$	$\frac{5^{5/4}}{5^{3/4}} = 5^{(5/4)-(3/4)} = 5^{1/2}$

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.335

Examples

- Solve $x^{2/3} = 16$
- Solve $2n^{-2} - n^{-1} = 3$
- Solve $x^{2/3} + 9x^{1/3} + 14 = 0$
- Solve $x - 5 = \sqrt{5x - 1}$
- Solve $\sqrt{2x - 4} + 2 = \sqrt{3x + 4}$